COLLECTIONS AND SEQUENCES OF CONTINUA IN THE PLANE

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1. An inversion of the plane with respect to a closed circular disc. The inversion described here will be used in proving Theorem 5.

DEFINITION. Let S denote the plane, let K be a closed circular disc, and let T be a one-to-one transformation of S onto itself satisfying the following conditions:

(1) T is continuous over K, and $T(K) = K$;

(2) T is continuous over $S-K$, and $T(S-K) = S-K$; and

(3) if H is an unbounded subset of $S-K$ which does not have a limit point in K, then $T(H)$ is bounded and has a limit point in K.

The transformation T will be called an *inversion of S with respect to K.*

NOTATION. If T is an inversion of the plane with respect to a closed circular disc K, and M is a continuum in $S - K$, then M' will denote the closure of $T(M)$. If G is a collection of continua in $S - K$, then G' will denote the collection of all continua X' such that X is a continuum of G. This notation will be used in the statement of Theorem 1 and in the proof of Theorem 5.

THEOREM 1. *If K is a closed circular disc and G is a finite collection of mutually exclusive unbounded continua not intersecting K, then there is an inversion T of the plane with respect to K such that the continua of G' are bounded and mutually exclusive.* [1]

Indication of proof. If the plane S is inverted about the boundary of K with respect to the center o of K, then the continua of G are carried onto mutually exclusive bounded connected sets each of which has o as a limit point and is closed relative to $S - o$. Hence it will be sufficient to show that if $M_1, M_2, \cdots, M_n$ $(n > 1)$ are bounded continua such that

[1] I am indebted to the referee for some very helpful suggestions which enabled me to obtain a simplified proof of this theorem.

Received September 29, 1953. Presented to the American Mathematical Society, May 2, 1953.

Pacific J. Math. 5 (1955), 325-333