THE NORM FUNCTION OF AN ALGEBRAIC FIELD EXTENSION, II

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1. Introduction. In our previous paper [3], we consider the general norm

$$N_{K/k}(\omega_1 X_1 + \cdots + \omega_n X_n)$$

of a finite extension K of an algebraic field k. We proved that this form is the (n/m)th power of an irreducible polynomial in $k[X]$, where m is the maximum of the degrees of the simple subfields $k(\theta)$ of K over k. The proof of this result used a considerable amount of the heavy machinery of the theory of algebraic extensions: the maximal separable subfield, conjugates, transitivity of the norm, etc. Using only the fact that the general norm is a power of an irreducible, we obtained a characterization of the norm function $N_{K/k}$ in terms of inner properties.

In the present paper we shall approach these matters from a different point of view. We shall give an entirely different proof that the general norm is a prime power—this one based on very little field theory and completely rational. From this, as noted above, the intrinsic characterization of the norm function follows. We shall then use this to derive certain theorems in field theory, such as the transitivity of the norm.

Section 2 contains some preliminary results on polynomials and their norms and the details of proof for certain results used in [3]. In §3 we prove the main result and in §4 we give some applications.

2. Tool theorems. We shall be dealing with polynomial rings $k[X]$ in indeterminates $X = (X_1, \cdots, X_r)$ and shall take for granted the fundamental fact that such rings are unique factorization domains [1, p. 39]. The following is well known, but we include it—as we do several of the results of this section—for completeness.

LEMMA 1. *Let $f(X)$, $g(X) \in k[X]$ and suppose f and g are relatively prime. Let $k \le K$ so that $k[X] \le K[X]$. Then f and g are still relatively prime when considered as elements of the extended ring $K[X]$.*

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