SOME REMARKS ON p-RINGS AND THEIR BOOLEAN GEOMETRY

JOSEPH L. ZEMMER

Introduction. In this paper the word *ring* will always mean a ring with identity, and the Boolean algebra associated with a Boolean ring B will mean the Boolean algebra corresponding to B in the one-to-one correspondence, described by Stone [10], between the set of all Boolean rings and the set of all Boolean algebras. In a Boolean algebra, $\cap$, $\cup$, $'$, will denote the operations of intersection, union, and complementation respectively.

A commutative ring R will be called a *Boolean valued ring* if there exists a Boolean algebra $\mathfrak{B}$, and a single valued mapping $x \rightarrow \phi(x)$ of R into $\mathfrak{B}$ satisfying:

(i) $\phi(x)=0$ if and only if $x=0$,

(ii) $\phi(xy)=\phi(x) \cap \phi(y)$,

(iii) $\phi(x+y) \subseteqq \phi(x) \cup \phi(y)$.

When such a mapping exists it will be called a *valuation* for R. It is not difficult to show that a ring is a Boolean valued ring if and only if it is isomorphic to a subdirect sum of integral domains. Hence every commutative regular ring is Boolean valued.

In a Boolean valued ring the function $d(x, y)=\phi(x-y)$ satisfies the usual requirements for a distance function, except that the "distance" is an element of a Boolean algebra. The investigation of the geometric properties of a Boolean ring with respect to the distance function defined above was begun by Ellis [3, 4] and has been extended by Blumenthal [1]. The present paper is mainly concerned with extending some of these results to a larger class of Boolean valued rings, namely the p-rings.

It seems that p-rings were first defined and studied by McCoy and Montgomery [7] in order to generalize the well known theorem of Stone on the structure of Boolean rings. In [7] it is shown that every p-ring is a subdirect sum of fields I_p. In any commutative ring R the idempotents form a Boolean ring with respect to the multiplication of

Received April 25, 1954, and in revised form Nov. 15, 1954. Presented to the American Mathematical Society, April 30, 1954. Most of the work contained in this paper was made possible by a University of Missouri Summer Research Fellowship held by the author in 1953. The author wishes to express his appreciation to the referee for making several substantial improvements, and to L. M. Blumenthal for many hours of stimulating conversation in connection with the paper.