SOME STRUCTURE THEOREMS FOR A CLASS OF BANACH ALGEBRAS

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0. Introduction. The purpose of this paper is the presentation of a structure theory for a class of Banach algebras which we define below and call *GS*-algebras. This class includes the commutative regular *B*-algebras of Silov [9] and many of our results generalize theorems and techniques of that author. In addition, several interesting types of non-commutative *B*-algebras (listed in § 1) which have been studied previously only individually and from rather widely deffering points of view are included in the class of *GS*-algebras. In § 1 we introduce some basic definitions and prove several fundamental theorems. § 2 contains some theorems on the structure of closed two-sided ideals in certain *GS*-algebras, and in § 3 we present a decomposition theory for such algebras.

1. Preliminary definitions and results. It is assumed that the reader is familiar to a certain extent with the theory of rings and ideals and the basic theory of *B*-algebras. The theory of regular commutative *B*-algebras can be found in [5], [6], or [9]. In this paper *ideal* will mean *two-sided ideal*. Consider a *B*-algebra R with structure space $S(R)$. $S(R)$ is the collection of maximal regular ideals of R with the standard Stone-Jacobson topology which is defined as follows: the closure F^c of a set $F \subset S(R)$ is $\{M \in S(R) \mid M \supset \bigcap M',\ M' \in F\}$. The terminology is rendered somewhat more manageable by defining the *kernel* $k(F)$ of a set F in $S(R)$ to be the intersection of all maximal regular ideals in F and the *hull* $h(I)$ of an ideal I in R to be the set of all elements of $S(R)$ which contain I. Then the hulls are the closed sets in $S(R)$. If $F=h(I)$ we say that I *belongs to* F. $S(R)$ is, in general, a T_1-space and it is compact if R contains an identity. We say that R is strongly-semi-simple (s.s.s.) if the intersection of all maximal regular ideals is zero.

If R has an identity then the theory we present below can be car-

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