ON LOCALLY COMPACT TOTALLY DISCONNECTED ABELIAN GROUPS AND THEIR CHARACTER GROUPS

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1. Introduction. The Pontrjagin duality theory [4] establishes a dual relationship between pairs of locally compact Abelian groups (referred to in the remainder of the paper as L.C.A. groups): Namely, if two groups are duals each is isomorphic and homeomorphic to the group of all continuous homomorphisms (character group) of the other into the circle K, the character group being provided with the compact-open topology. The question arises whether this same property will hold true for groups of mappings into some group other than K. Pontrjagin answers this in the negative by showing that if we consider mappings into any group other than K, that K itself will not be the character group of its character group. However, for a more restricted class of groups than the class of all L.C.A. groups, the duality property might yet hold true for mappings into some group other than the circle. The purpose of this paper is to show that the duality theorem is valid if we consider mappings of any L.C.A. totally disconnected (L.C.T.D.A.) group into the discrete rationals modulo 1. It is shown that the same theorem is true using various topologies on this group and a necessary and sufficient condition on the topology is given in order that the theorem be true. For the case of L.C.A. p-primary groups, it is shown that the duality theorem is true using *any* separated topology on the p-primary component of the rationals modulo 1. Since the rationals modulo 1 are a subgroup of the circle, we are merely using different topologies on the necessary part of the circle, the resulting character groups being the same as those in the classical case. However, for the case of compact T.D.A. groups, a genuinely new theory is exhibited in which the character groups are not in general the same as those of the Pontrjagin theory. Throughout the paper the word *group* will be used to denote an Abelian separated topological group. The additive notation will be used for the group operation. We shall use the symbol $\simeq$ to represent an algebraic isomorphism and $\cong$ to represent an algebraic isomorphism *and* topological homeomorphism.

2. Character theory.

DEFINITION. Let G and g be two groups. g will be called the

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