ON THE NUMBER OF ABSOLUTE POINTS OF A CORRELATION

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1. Introduction. In 1948, R. W. Ball [2] presented methods for obtaining information about the number of absolute points of a correlation of a finite projective plane in which neither the theorem of Desargues nor any other special property (except, of course, the existence of the correlation) is assumed. This work was, in a sense, a continuation of an earlier investigation by R. Baer [1] of the case that the correlation is a polarity.

We shall show how, using an incidence-matrix approach[1], one may obtain the principal results of [2] somewhat more directly. Some of the results are strengthened. In addition, our method is sufficiently general to apply at once to the so-called symmetric group divisible designs, a class of combinatorial configurations including the finite projective planes. For simplicity, we shall present our main discussion in the language of planes, reserving to the end indications of the generalization.

As pointed out in §§ 3 and 4 the geometric problem with which we are concerned leads naturally to the question: What are the irreducible polynomials whose roots are roots of natural numbers? This question is treated in the following section.

2. Polynomials whose roots are roots of natural numbers. Let $f(x)$ be an irreducible polynomial with integral coefficients and let one of its roots be $z=n^{1/k}\zeta$, (n, k natural numbers, ζ a root of unity). Clearly z satisfies the equation

$$z^k/n=\zeta^k=\zeta_h \tag{1}$$

for some h, where from now on we use ζ_h to denote a primitive hth root of unity. From (1) we see that $\Phi_h(z^k/n)=0$, where Φ_h is the cyclotomic polynomial of order h. Hence

$$f(x)\,|\,n^{\varphi(h)}\Phi_h(x^k/n)\ . \tag{2}$$

The problem is therefore reduced to that of finding the irreducible factors of $\Phi_h(x^k/n)$ for arbitrary positive integers h, k, n. It will suffice

Received August 16, 1954. The work of the first two authors was supported (in part) by the Office of Naval Research.

[1] Arithmetic properties of the incidence matrix have been exploited with conspicuous success ([4], [5]). In this paper we study its characteristic polynomial.