COMPLEX BUNDLES WITH ABELIAN GROUP

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We shall be concerned here with certain complex bundles whose bundle groups are Abelian and with some bundle spaces of such bundles, when a particular fibre is employed. We begin with a general definition of complex bundle and attempt to describe some of the work of Kodaira and Spencer in this context. A general theory is then described. This is followed by considering a special case, the group being that of a one dimensional Abelian variety, that is, the additive group of complex numbers reduced modulo a discrete subgroup which is generated by two complex numbers whose ratio is not real. Certain efforts toward an explicit classfication of such bundles are made, together with an attempt to relate the concepts here introduced to some in the Kodaira-Spencer theory, and to some more classical notions. A version of an Abel's theorem is given.

The last section of the paper is related to the "general theory", mentioned above, only to the extent that the spaces considered are the bundle spaces of bundles with an Abelian group. Most of the results and statements in the last section are not dependent on the earlier sections of the paper, and are perhaps related to the earlier sections only in the author's mind, because they arose at the same time and from the same considerations. The principal result there is a construction, involving Hopf manifolds, which shows that Kählerian structure is very rare among compact complex manifolds.

1. Complex bundles and the Kodaira-Spencer theory. Let V be a complex manifold, $\mathscr{U}=\{U_i\}$ an indexed covering of V by coordinate neighborhoods, $N(\mathscr{U})$ the nerve of the covering $\mathscr{U}$, G a complex Lie group, $\mathfrak{G}$ the sheaf of germs of complex analytic mappings of V into G. Suppose analytic mappings $\theta_{ij}: U_i \cap U_j \to G$ are given, such that

$$\theta_{ij}(x)\cdot\theta_{jk}(x)\cdot\theta_{ki}(x)=\text{identity}, \qquad \text{for } x\in U_i\cap U_j\cap U_k,$$

$$\theta_{ii}(x)=\text{identity}, \qquad \text{for } x\in U_i.$$

We say the set $\{\theta_{ij}\}$ defines a $\mathscr{U}$-coordinate bundle with respect to G, or merely a $\mathscr{U}$-bundle. The θ_{ij} are called the coordinate transformations. Two $\mathscr{U}$-bundles $\{\theta_{ij}\}$ and $\{\theta'_{ij}\}$ are said to be $\mathscr{U}$-equivalent (with respect to G) if there exist analytic mappings $\lambda_i: U_i\to G$, such that

$$\theta'_{ij}=\lambda_i\theta_{ij}\lambda_j^{-1} \qquad \text{for } x \text{ in } U_i\cap U_j.$$

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