ON THE NUMBER OF DISSIMILAR LINE-SUBGRAPHS OF A GIVEN GRAPH

FRANK HARARY

1. Introduction. Two enumeration formulas are obtained in this paper. The first provides a solution for the counting polynomial which gives the number of dissimilar line-subgraphs of a given graph, and is a generalization of the formula due to Pólya for the number of graphs which appears in [2]. The second serves to find the number of graphs in which a prescribed subgraph is distinguished or rooted. The special case in which a single point of the graph is distinguished is called a *rooted graph* [3, p. 76]. The number of rooted graphs also appears in [2].

Both of these results utilize in an essential way the classical enumeration theorem of Pólya [4] which enables one to express the configuration-counting series in terms of the configuration group and the figure-counting series. In order that these results be self-contained, we briefly review Pólya's Theorem, specialized to one variable. Let *figure* be an undefined term. To each figure there is assigned a non-negative integer called its *content*. Let φ_k denote the number of different figures of content k. The figure-counting series $\varphi(x)$ is defined by

$$\varphi(x)=\sum_{k=0}^{\infty}\varphi_k x^k.$$

Let Γ be a permutation group of degree s and order h. A *configuration* of length s is a sequence of s figures. The content of a configuration is the sum of the contents of its figures. Two configurations are *Γ-equivalent*, if there is a permutation of Γ sending one into the other. Let F_k denote the number of Γ-inequivalent figures of content k. The configuration-counting series $F(x)$ is defined by

$$F(x)=\sum_{k=0}^{\infty}F_k x^k.$$

The permutation group Γ will be called the *configuration group*.

Pólya's Theorem expresses $F(x)$ in terms of $\varphi(x)$ and Γ, using the cycle index of Γ. Let $h_{j_1 j_2 \cdots j_s}$ be the number of elements of Γ of type $(j_1, j_2, \cdots, j_s)$, that is, having j_k cycles of length k, $(k=1, \cdots, s)$ so that

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