A NOTE ON ABSTRACT MEASURE

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1. Introduction. It has been shown by Segal ([1], our definitions follow [1] whenever possible, except for our introduction of condition (4) into the definition of a measure space) that localizable measure spaces form the largest class of measure spaces to which certain processes of functional analysis may be applied. These measure spaces are characterized as those which are strongly equivalent to direct sums of finite measure spaces. It has some interest to ask under what conditions these finite measures may be obtained as subsets of the original measure space. We shall show that this is always possible provided the 'dimension' of the measure space is equal to or less than the power of the continuum. If one assumes the continuum hypothesis, then every measure space satisfying the given condition on its 'dimension' is localizable in this stronger sense.

2. Definitions. We shall fix a *measure space*, that is, a triple $M=\langle R, \mathfrak{R}, r\rangle$, where

(1) $\mathfrak{R}$ is a non-void family of subsets of a set R, closed under relative complementation and finite union;

(2) r is a nonnegative finite-valued finitely additive function on $\mathfrak{R}$;

(3) r is countably additive in the restricted sense that, if $\{E_n\}$ is a disjoint sequence of sets in $\mathfrak{R}$ such that

$$\sum_{n=1}^{\infty} r(E_n)=s<\infty ,$$

then

$$\bigcup_{n=1}^{\infty} E_n \in \mathfrak{R} \text{ and } r(\bigcup_{n=1}^{\infty} E_n)=s .$$

We shall also make the following inessential restriction:

(4) If $E\in\mathfrak{R}$, $r(E)=0$, $A\subset E$, then $A\in\mathfrak{R}$.

A subset A of R is *measurable* resp. *null* if

$$A\cap E\in\mathfrak{R} \text{ resp. } r(A\cap E)=0$$

for all E in $\mathfrak{R}$. Two subsets A and B of R are *equivalent* if $A\ominus B$ ($\ominus$ is symmetric difference) is a null set, and *almost disjoint* if $A\cap B$ is a null set.

The σ-Boolean algebra of all measurable sets modulo null sets is the *measure ring* of M. M is *localizable* if the measure ring of M is complete. By the *dimension* of M we shall mean the smallest cardinal

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