ON THE NUMERICAL INTEGRATION OF QUASI-LINEAR PARABOLIC DIFFERENTIAL EQUATIONS

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1. Introduction. The following differential equation will be considered in the region $0 \leqq x \leqq 1$, $t \geqq 0$:

$$\frac{\partial^2 u}{\partial x^2} = F(x, t, u)\frac{\partial u}{\partial t} + G(x, t, u), \qquad F \geqq m > 0. \tag{1.1}$$

Physical phenomena leading to equations of this type include heat conduction problems in which the thermal diffusivity depends on both position and temperature, certain diffusion problems, and the flow of compressible single-phase fluids through porous media.

The simplest example of (1.1) is the classical heat flow equation

$$\frac{\partial^2 u}{\partial x^2} = K\frac{\partial u}{\partial t}, \qquad K \text{ constant},$$

which describes the flow of a fluid of constant compressibility in a linear reservoir as well as the conduction of heat in a bar insulated except possibly at its ends. If either of these problems is considered in an annular region in which radial symmetry exists, then the equation

$$\frac{\partial^2 u}{\partial x^2} = Ke^{2x}\frac{\partial u}{\partial t}$$

applies, where x is to be interpreted as the logarithm of the radius.

A somewhat more complex example is furnished by the linear flow of an ideal gas. In this case,

$$\frac{\partial^2 u}{\partial x^2} = \frac{K}{2u^{1/2}}\,\frac{\partial u}{\partial t},$$

where $u = p^2$, p being the pressure. The effect of treating real gases rather than ideal is that the coefficient of u_t becomes more involved.

Diffusion problems involving chemical reactions often may be analyzed by studying equations of the type

$$\frac{\partial^2 u}{\partial x^2} = K\frac{\partial u}{\partial t} + g(x, t, u).$$

As each of the examples cited are special cases of the general equa-

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