QUOTIENT ALGEBRA OF A FINITE AW*-ALGEBRA

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1. Introduction. In a recent paper [5] Wright proves that *if A is an AW^*-algebra* [2] *having a trace and if M is a maximal ideal of A, then A/M is an AW^*-factor* (that is, *an AW^*-algebra whose center consists of complex numbers*) *having a trace.* The trace enters into his argument in the characterization [5, Theorem 3.1] of the one-to-one correspondence between maximal ideals of A and those of its center Z. This is, in turn, used to verify that A/M satisfies the countable chain condition, namely: every set of mutually orthogonal projections is at most countable, which is crucial to prove that every set of mutually orthogonal projections has a least upper bound (LUB). It is the purpose of this paper to prove the following.

THEOREM. *Let A be a finite AW^*-algebra, and M a maximal ideal of A. Then A/M is a finite AW^*-factor.*

It is not known whether a finite AW^*-factor always has a trace. Since [3] a finite AW^*-algebra of type I always has a trace, our result adds nothing new in this case, and we shall be solely concerned with algebras of type II_1.

Our terminology is that of [2]. We assume familiarity with [2] and [1] (especially [1, pp. 234–242]).

2. Maximal ideal M. We begin with a slightly sharpened version of [5, Theorem 2.5] on p-ideals. A set P of projections is called a p-ideal if

(1) P contains $e\vee f$ whenever it contains e and f

(2) P contains f whenever it contains an $e\succ f$.

It follows from (1) that $e_1\vee\cdots\vee e_n$ is in P if $e_1,\cdots,e_n$ are in P. For any set S of A let S_p denote the set of projections contained in S.

LEMMA 1. *Let A be an AW^*-algebra. The closed linear subspace M generated by a p-ideal P is an ideal with $M_p=P$. Conversely an ideal M of A is the closed linear subspace generated by the p-ideal M_p.*

Proof. Let P be a p-ideal and M the closed linear subspace generated by P. For M to be an ideal we need to prove that M contains xe for any $x\in A$ and $e\in P$. The left projection [2, p. 244] f of xe, being $\prec e$, is contained in P. Hence P contains $g=e\vee f$. $xe\in gAg\subset M$,

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