ASYMPTOTIC RELATIONS BETWEEN SYSTEMS OF DIFFERENTIAL EQUATIONS

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1. Introduction. A. Wintner [1], N. Levinson [2], H. Weyl [3] and others have obtained interesting asymptotic relations between the solutions of a given system of differential equations and those of an approximate system. In their investigations the solutions of the approximate system of differential equations are assumed to be bounded. In this paper we consider the asymptotic problems of the solutions from a more general point of view, given only certain order relations satisfied by the solutions of the approximate system. The method we shall use is to study an associated system of integral equations which yields the asymptotic relations between the solutions of the perturbed and unperturbed equations. With the aid of the Phragmén-Lindelöf Theorems [4], our results can be easily extended to the complex domain.

2. Asymptotic relations in the real domain. Consider the system of differential equations written in the vector form

$$\frac{dy}{dx}=A(x)y+f(x, y) , \tag{2.1}$$

where $A(x)=\|a_{ij}(x)\|$ is a $n\times n$ matrix and y and $f(x, y)$ are respectively column vectors with components $y^{(i)}$ and $f^{(i)}(x, y)$, $i=1, 2, \cdots, n$. Defining the norms $\|y\|$, $\|A\|$ of vectors y and matrices A by

$$\|y\|=\sum_{i=1}^{n}|y^{(i)}|, \quad \|A\|=\sum_{i,j=1}^{n}|a_{ij}| ,$$

it is easy to verify that

$$\|y_1+y_2\|\leqq\|y_1\|+\|y_2\| , \qquad \|A_1+A_2\|\leqq\|A_1\|+\|A_2\| ,$$
$$\|A_1A_2\|\leqq\|A_1\|\|A_2\| , \qquad \|Ay\|\leqq\|A\|\|y\| .$$

In this section we assume that $a_{ij}(x)$ and $f^{(i)}(x, y)$ (for each fixed complex y) are complex-valued functions of the real variable x belonging to $L(0, R)$ for every positive R. Furthermore we assume that for each $x\geqq 0$, $f^{(i)}(x, y)$ is a continuous function of y for all complex y and $f(x, y)$ satisfies the Lipschitz condition

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