AN INTRINSIC INEQUALITY FOR LEBESGUE AREA

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1. Introduction. An inequality for Lebesgue area which corresponds to the fact that the measure of a quadrilateral is not less than the product of the distances between the two pairs of opposite sides may sometimes be useful for the study of this area. This inequality is an extension of a result of Besicovitch [2].

The important results of Cesari [4] and Federer [8] showing the equivalence of Geöcze and Lebesgue area will be used to show that several other 'areas' are equivalent to these two.

This paper depends upon definitions and results of [11] and [12]. In particular we shall use the area defined in [11] which agrees with Lebesgue area for surfaces in Euclidean space.

Let Q be the square $0 \leqq u, v \leqq 1$ having consecutive sides a, b, c, and d. The set of continuous functions on Q into m, the space of bounded sequences [1], will be denoted by C, and the family of homeomorphisms of Q into Q by H.

Let $x, y \in C$. Then $\tilde{x}$ is defined on $Q \times Q$ to the nonnegative real numbers by

$$\tilde{x}(p, q) = ||x(p) - x(q)||$$

for $(p, q) \in Q \times Q$. If there exists a positive real number M such that

$$\tilde{x}(p, q) \leqq M||p - q||$$

for all $(p, q) \in Q \times Q$ then x is *Lipschitzian*. If $\tilde{x}(p, q) \leqq \tilde{y}(p, q)$ for all $(p, q) \in Q \times Q$, then we shall write $\tilde{x} \leqq \tilde{y}$. The Lebesgue area of x is denoted by $L(x)$. If $\tilde{x} \leqq \tilde{y}$ then $L(x) \leqq L(y)$ [Kolmogoroff's principle].

If i and k are distinct positive integers, let π_{ik} be the plane in m consisting of those points all of whose components, except the ith and kth, are zero. The set of all planes π_{ik} is Π. Let E^2 be the Euclidean plane provided with a cartesian coordinate system, and let T_{ik} be the homeomorphism of E^2 onto π_{ik} defined by

$$T_{ik}(s, t) = \{w^j\} ,$$

where $(s, t) \in E^2$, $w^i = s$, $w^k = t$, and $w^j = 0$ for $i \neq j \neq k$. If $E \subset E^2$, and E is Lebesgue measurable, then the measure of $T_{ik}(E)$, $|T_{ik}(E)|$, is the

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