SOME INTEGRAL FORMULAS FOR CLOSED HYPERSURFACES IN RIEMANNIAN SPACE

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Introduction. Let V^n be a hypersurface twice differentiably imbedded in a Riemannian space R^{n+1} of $n+1$ ($n \geqq 2$) dimensions, and $\kappa_1, \cdots, \kappa_n$ the n principal curvatures at a point P of the hypersurface V^n. It is known that the ith mean curvature M_i of the hypersurface V^n at the point P is defined by

$$C_{n,i}M_i = \sum \kappa_1\kappa_2\cdots\kappa_i \qquad (i=1, \cdots, n), \tag{0.1}$$

where the expression on the right side is the ith elementary symmetric function of $\kappa_1, \cdots, \kappa_n$, and $C_{n,i}$ denotes the number of combinations of n different things taken i at a time. Let dA be the area element of the hypersurface V^n at the point P, and p the scalar product of the unit normal vector of the hypersurface V^n at the point P and the position vector of the point P with respect to any orthogonal frame in the space R^{n+1}.

The purpose of this paper is to prove the following four theorems concerning closed hypersurfaces by first showing that:

a) If V^n is an orientable hypersurface, with a closed boundary V^{n-1} of dimension $n-1$ ($n \geqq 2$), which is twice differentiably imbedded in an $(n+1)$-dimensional Riemannian space R^{n+1}, then the integral $\int_{V^n}(1+M_1p)dA$ can be expressed as an integral over the boundary V^{n-1}.

b) If in addition V^n is of class C^3 and the space R^{n+1} is of constant Riemannian curvature, then the integral $\int_{V^n}(M_{n-1}+M_np)dA$ can also be expressed as an integral over V^{n-1}.

These results have been obtained in a previous paper [2] by the author for an orientable hypersurface V^n twice differentiably imbedded in a Euclidean space E^{n+1} of $n+1$ ($n \geqq 2$) dimensions.

THEOREM 1. *Let V^n be a closed orientable hypersurface twice differentiably imbedded in a Riemannian space R^{n+1} of $n+1$ ($n \geqq 2$) dimensions, then*

$$A+\int_{V^n} M_1 p dA = 0 \,. \tag{I}$$

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