ON THE UNIFORM CONVERGENCE OF A CERTAIN EIGENFUNCTION SERIES

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1. Introduction. In the attempt to solve certain problems in mathematical physics, such as diffraction of an arbitrary pulse by a wedge as considered by Irvin Kay [1], one encounters a hyperbolic differential equation of the type

$$u_{xx}-q(x)u=u_{xt}-p(x)u_t \tag{a}$$

where $u(x, t)$ must satisfy the boundary conditions $u(1, t)=u(0, t)=0$ and $u(x, 0)=F(x)$. In attempting to solve equation (a) by separation of variables, one is led to the consideration of expanding an arbitrary function $F(x)$ in terms of the eigenfunctions $u_n(x)$ of the equation

$$u''+q(x)u+\lambda(p(x)u-u')=0$$

satisfying the boundary conditions $u(0)=u(1)=0$.

In the previous paper [2] by B. Friedman and L. I. Mishoe, it was proved that a function $F(x)$ of bounded variation for $0\leqq x\leqq 1$ could be expanded in terms of the eigenfunctions $u_n(x)$ of the system $u''+qu+\lambda(pu-u')=0$, $u(0)=u(1)=0$, provided $F(0^+)+F(1^-)\exp\left(-\int_0^1 pdt\right)=0$. However, the question of uniform convergence of the series $\sum_{-\infty}^{\infty} a_n u_n(x)$ to $F(x)$ was not considered. In this paper we establish sufficient conditions for the series $\sum_{-\infty}^{\infty} a_n u_n(x)$ to converge uniformly to $F(x)$ for $0<x<1$.

The following theorem has already been proved [2]:

THEOREM 1. *Let $F(x)$ be a function of bounded variation for $0\leqq x\leqq 1$. Let $u_n(x)$ be the eigenfunctions of the system*

$$(A+\lambda B)u=0\ ; \qquad u(0)=u(1)=0\ , \tag{1}$$

where A is the operator $d^2/dx^2+q(x)$, and where B is the operator $-d/dx+p(x)$.

Let $q(x)$ be continuous and $p(x)$ have a continuous second derivative. Furthermore, let $v_n(x)$ be the eigenfunctions of the system adjoint to (1). If

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