REMARK ON THE USE OF FORMS IN VARIATIONAL CALCULATIONS

LOUIS AUSLANDER

In [1] we introduced in equation (2.2) the form

$$\omega=\sum \frac{\partial F}{\partial q_i'} dq_i-\Big(\sum q_i' \frac{\partial F}{\partial q_i'}-F\Big)dt .$$

The purpose of this note is to explain the reason for introducing precisely this form.

We considered the integral

$$I=\int_a^b F(q_1, \cdots, q_n; q_1', \cdots, q_n'; t)dt .$$

Now let S be the set of all forms η such that if X denotes the vector field to a curve C with equations

$$q_i=q_i(t)$$

$$q_i'=\frac{dq_i}{dt}$$

$$t=t$$

Then $\langle X, \eta\rangle=F$ or $I=\int_a^b\langle X, \eta\rangle dt$. The set S is certainly not void since $F(q_1, \cdots, q_n; q_1', \cdots, q_n; t)dt$ and ω are contained in it. We will prove the following.

THEOREM. *There exists one and only one form ω in S such that along every curve of the above type ω and $d\omega$ give rise to forms in the space $(q_1, \cdots, q_n, t)$.*

Proof. The hypotheses of this theorem are equivalent to the following two analytic conditions:

1. $\langle\partial/\partial q_i', \omega\rangle=0$
2. $\langle\partial/\partial q_i'\wedge\partial/\partial x, d\omega\rangle=0$ when $q_i'dt=dq_i$ where x is any of the coordinates (q_i, q_i', t), $i=1, \cdots, n$.

Condition 1. implies that $\omega=\Sigma a_i dq_i+bdt$. Now since $\omega\in S$ we must have

Received February 17, 1956. This paper was written while the author was a National Science Foundation Postdoctoral Fellow.