ON DISTORTION IN PSEUDO-CONFORMAL MAPPING

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1. Introduction ; the method of the minimum integral. One aim of the theory of functions of several complex variables is to reformulate methods of the theory of conformal mappings in such a way that these methods can be successfully applied to obtain results in the theory of pseudo-conformal mappings, that is, in mappings of domains of the $(z_1, \cdots, z_n)$-space by n analytic functions of the n complex variables $z_1, \cdots, z_n$.[1] The determination of bounds for the distortion of Euclidean measures under pseudo-conformal transformation is one of the main topics of this branch of the theory.

An important tool in investigations of this kind is Bergman's method of the minimum integral [3, p. 48]. The basic idea is as follows. After an invariant[2] (non-Euclidean) metric is introduced in a domain B, *the ratios of the non-Euclidean and the Euclidean measures of geometric objects in B are expressed in terms of quantities λ_B which are solutions of the minimum problems*:

$$\lambda_B = \min \int_B |f|^2 d\omega \ . \tag{1.1}$$

Here f is an analytic function, regular in B and is subjected to certain auxiliary conditions[3], and $d\omega$ is the element of volume (the element of area in the case of one complex variable). Because of the specific choice of the auxiliary conditions, these λ_B possess the property that they *are monotone functions of the domain B*, that is if $B_1 \supset B$ then $\lambda_{B_1} \geqq \lambda_B$. As a rule λ_B can be expressed in terms of Bergman's kernel functions of B and its derivatives and thus can be calculated for special domains. These λ_B are of much interest because they can be easily applied to obtain distortion theorems ; for instance, if $I \subset B \subset A$, where I and A are domains for which the kernel functions $K^I(z, \bar{z})$ and $K^A(z, \bar{z})$ can be expressed in a closed form[4], then $\lambda_I \leqq \lambda_B \leqq \lambda_A$ and λ_I, λ_A are known quantities. With

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[1] In the present paper we consider only the case of two complex variables, $n=2$. However, it should be stressed that the methods used here can be easily generalized to the case of n variables, $n > 2$. The additional difficulties which arise are of a purely technical nature.

[2] Invariant with respect to pseudo-conformal transformation.

[3] By varying the auxiliary conditions, one obtains different λ_B's. As a rule upper indices on λ_B indicate the auxiliary conditions. For details see § 2.

[4] In such case we refer to I and A as "domains of comparison".