FUNCTIONALS ASSOCIATED WITH A CONTINUOUS TRANSFOMATION

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1. Let T: $z=t(w)$, $w \in R_0$, be a continuous transformation from a simply connected polygonal region R_0, in the Euclidean plane π, into Euclidean three-space. The transformation T is a representation for an F-surface of the type of the 2-cell in Euclidean three-space, which will be called, in brief, a surface S. [4, II. 3.7, II. 3.44].

In connection with transformation T, T. Radó defines a non-negative (possibly infinite) functional $a(T)$, which he shows is independent of the representation T for the surface S. [4, V. 1.6]. Radó calls $a(T)$ the lower area of the surface, and it plays an important role in the study of surface area.

P. V. Reichelderfer has also defined a non-negative (possibly infinite) functional $eA(S)$, which he calls the essential area of the surface S. [5, p. 274]. It too is an important concept in surface area theory.

The question arises as to what relationship exists between the lower area $a(T)$ and the essential area $eA(S)$. In this paper, we show that $eA(S) = a(T)$. In addition, we introduce certain other functionals, which we show yield the same value as that of $eA(S)$ and $a(T)$. These functionals, as well as $eA(S)$ and $a(T)$, will be defined in § 3, after a discussion in § 2 of necessary topological concepts.

2. Let M be a metric space. If $A \subset M$, then $M-A$, $\mathrm{c}(A)$, $\mathrm{i}(A)$, and $\mathrm{fr}(A)$ denote respectively, the complement, closure, interior, and frontier of A. If $A \subset M$, $B \subset M$, then $A \cup B$, $A \cap B$, and $A-B$ denote the union, intersection, and difference of A and B. ϕ denotes the empty set. If $\{A_n\}$ is a sequence of subsets of M, then $\bigcup_{n=1}^{\infty} A_n$ and $\bigcap_{n=1}^{\infty} A_n$ denote respectively the union and intersection of these sets.

Let F: $z=f(w)$, $w \in M$, be a continuous transformation from a metric space M into a metric space N. If $P \subset M$, the symbol $F|P$ denotes the transformation F with its domain restricted to P.

If $z \in N$, let $(F|P)^{-1}z$ denote the set of points w such that $w \in P$, $f(w)=z$. If $(F|P)^{-1}z \neq \phi$, then the components of $(F|P)^{-1}z$ are called maximal model components for z under $F|P$. If a maximal model component for z under $F|P$ is a continuum, then it is called a maximal model continuum (henceforth abbreviated m.m.c.) for z under $F|P$.

Now let F: $\bar{z}=f(w)$, $w \in R_0$, be a continuous transformation from a

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