MINIMAL DOMAINS AND THEIR BERGMAN KERNEL FUNCTION

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1. Introduction. On certain problems which arise in the theory of conformal and pseudo-conformal transformations. Attempts to generalize the Riemann mapping theorem to the case of multiply connected domains lead to the concept of canonical domains, that is, domains of a special geometric "shape" (for example, the entire plane slit along parallel rectilinear lines), onto which any other domain can be mapped conformally. The existence of such mappings, though interesting in itself, is less useful than the Riemann mapping theorem, because the circle has many properties other canonical domains do not possess. Moreover, this situation is worse when one passes to the space of several complex variables. In fact, even in the case of a simply connected domain, a complete set of canonical domains having a well defined geometric shape is not known. If we return to the case of a plane domain and note that the existence of canonical domains can be proved by considering certain extremum problems, we are led to the conclusion that perhaps it is worthwhile to introduce other types of canonical domains, *defined by a simpler extremum problem* rather than by geometric shape. Indeed, two important types of such domains were investigated by S. Bergman: The *minimal domains* and the *representative domains*. (Bergman [2, pp. 27–42]). The shape of these domains (for definition, see § 2, 6.) is in general complicated, often they are situated on non-univalent Riemann hypersurfaces, but they possess many properties which enable us to deduce interesting results in the theory of pseudo-conformal transformations.

In the last century, various domain-functions were introduced and applied successfully to all branches of analysis. It proved useful to consider two types of problems: 1. Given some information on a domain—to find estimates for one of its domain functions. 2. Given properties of a domain function belonging to some domain—to obtain information about the domain itself. Theorems of these types lead, for instance, to distortion theorems and to solutions of various extremum

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