ON SOME SPECIAL SYSTEMS OF EQUATIONS

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1. Let F be an arbitrary field. Let S be a system of equations which, when solved for two of its variables, takes the following form:

$$x_1^{k_1}=f(x_3, \cdots, x_n)\,, \qquad x_2^{k_2}=g(x_3, \cdots, x_n)\,, \tag{1}$$

where f and g are arbitrary functions of the indicated variables. Consider also the equation

$$y^{k_1k_2}=f^{sk_2}(y_3, \cdots, y_n)g^{rk_1}(y_3, \cdots, y_n)\,. \tag{2}$$

THEOREM 1. *If* $(k_1, k_2)=1$ *and* $rk_1+sk_2=1$, *then the distinct solutions of* (1) *in* F *with* $x_1x_2\neq 0$ *may be put in one-to-one correspondence with the distinct solutions of* (2) *in* F *with* $y\neq 0$. *Moreover, these solutions of* (1), $x_1x_2\neq 0$, *may be determined from the solutions of* (2), $y\neq 0$, *and conversely, by means of transformations* (3) *and* (4) *below.*

Proof. Assuming for the rest of this section that $x_1x_2\neq 0$, $y\neq 0$, we put

$$x_1=y^{k_2}\left\{\frac{f(y_3, \cdots, y_n)}{g(y_3, \cdots, y_n)}\right\}^r, \qquad x_2=y^{k_1}\left\{\frac{g(y_3, \cdots, y_n)}{f(y_3, \cdots, y_n)}\right\}^s, \qquad x_i=y_i \quad (i=3, \cdots, n) \tag{3}$$

and notice that if $(y, y_3, \cdots, y_n)$ is a solution of (2) then (3) determines a solution of (1). Now let

$$y=x_1^s x_2^r\,, \qquad y_i=x_i \quad (i=3, \cdots, n)\,. \tag{4}$$

It may be verified directly that if $(x_1, x_2, \cdots, x_n)$ is a solution of (1) then (4) determines a solution of (2). Further, given a solution $(x_1, x_2, \cdots, x_n)$ of (1) and a solution $(y, y_3, \cdots, y_n)$ of (2) with $x_i=y_i$ $(i=3, \cdots, n)$, then (3) implies (4) and conversely—which may be verified with the use of the relation $rk_1+sk_2=1$.

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