NON-RECURRENT RANDOM WALKS

K. L. CHUNG AND C. DERMAN[1]

Introduction and Summary. Let $\{X_i\}$ $i=1, 2, \cdots$ be a sequence of independent and identically distributed integral valued random variables such that 1 is the absolute value of the greatest common divisor of all values of x for which $P(X_i=x)>0$. Define

$$S_n=\sum_{i=1}^{n} X_i .$$

Chung and Fuchs [5] showed that if x is any integer, $S_n=x$ infinitely often or finitely often with probability 1 according as $EX_i=0$ or $\neq 0$, provided that $E|X_i|<\infty$. Let $0<EX_i<\infty$, and A denote a set of integers containing an infinite number of positive integers. It will be shown that any such set A will be visited infinitely often with probability 1 by the sequence $\{S_n\}$ $n=1, 2, \cdots$. Conditions are given so that similar results hold for the case where X_i has a continuous distribution and the set A is a Lebesgue measurable set whose intersection with the positive real numbers has infinite Lebesgue measure.

A Theorem about Markov Chains. Let $\{Z_n\}$, $n=0, 1, \cdots$ denote a Markov chain with stationary transition probabilities where each Z_n takes on values in an abstract state space $\boldsymbol{X}$. The distribution of Z_0 is given but arbitrary. Let Ω denote the space of all possible sample sequences w, P the probability measure over Ω and $P(\cdot|\cdot)$ the conditional probability. The following theorem appears in [4].

THEOREM 1. *Let A be any event in $\boldsymbol{X}$. A sufficient condition that*

$$P(Z_n \in A \text{ infinitely often})=1 \tag{1}$$

is

$$\inf_{z\in X} P(Z_n \in A \text{ for some } n|Z_0=z)>0 . \tag{2}$$

Since [4] is not readily accessible, we shall prove the theorem here.

Proof.[2] We have with probability 1 that for $j\geq N$

Received April 23, 1955 and in revised form August 1, 1955. Work supported in part by the United States Air Force through the office of Scientific Research of the Air Research and Development Command under contract AF 18 (600)-760.

[1] Now at Columbia University.

[2] The proof given here is a modification of one suggested by J. Wolfowitz.