CONSTRUCTION OF THE LATTICE OF COMPLEMENTED IDEALS WITHIN THE UNIT GROUP

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In his book "Linear algebra and projective geometry" [1, pp. 203–227], R. Baer shows that in the ring of endomorphisms of a linear manifold, (F, A), except where the characteristic of F is 2, the projective geometry of the subspaces of the linear manifold is determined entirely within the multiplicative group of units in the ring. G. Ehrlich [2], using similar methods showed that the structure of a continuous geometry is determined within the unit group of the associated regular ring. The purpose of this paper is to show that a unified treatment may be given.

We will assume throughout that the ring R has an identity element which we denote by 1. We will say that a right ideal A in R is a *complemented right ideal* if there exists a right ideal A' such that $R = A \oplus A'$ where $\oplus$ indicates direct sum. We refer to such an ideal by the abbreviation C. R. I.

If K is any ring with identity, we denote the *unit group* of K by $U(K)$. Where K is R, this will be shortened to just U. For any set S of elements in R, we let $Z(S)$ denote the *center* of S, that is, the set of all those elements of S which commute with every element in S.

We assume the ring R satisfies the following postulates:

1. The mapping $r \to r+r$ for every element $r \in R$ is an automorphism of the additive group of R onto R. [1, p. 203; 2, p. 9]

This postulate requires a little more than that the characteristic of R is different from 2. We will denote $r+r$ by $2r$ and the inverse image of r by $\frac{1}{2}r$.

2. If A and B are C. R. I.'s then $A \cap B$ and $A \cup B$ are C. R. I.'s. [1, pp. 178, 179; 2, p. 6]

3. If e is a nonzero idempotent in R and if k is any element of R, then either $eRk=0$ or $kRe=0$ implies that $k=0$. [1, p. 198; 2, p. 16]

4. If e is an idempotent element of R, then $Z(U(eRe)) \leqq Z(eRe)$. [1, p. 201; 2, p. 14]

5. $Z(R)$ contains no nonzero divisors of zero. [1, p. 202; 2, p. 7]

An element of $u \in R$ is termed an *involution* if $u^2=1$. An element $s \in R$ which is the product of two distinct involutions and satisfies the property that $(s-1)^2=0$ is said to be of class two. Section 1 deals with

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