A CHARACTERIZATION OF W^*-ALGEBRAS

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1. Introduction. Many results have been obtained for W^*-algebras of bounded operators on a Hilbert space. However one of the most unsatisfactory parts of the present theory of W^*-algebras is the dependence on the underlying Hilbert space.

I. Kaplansky [5] has given important developments for the removal of this difficulty, but it is now known that his AW^*-algebra is not necessarily a W^*-algebra [cf. 4].

On the other hand, J. Dixmier [3] showed that a W^*-algebra is an adjoint space, and Z. Takeda has given a kind of characterization of W^*-algebras in [10].

The purpose of this paper is to give a space-free characterization in the following theorem:

THEOREM. *A C^*-algebra is a W^*-algebra if and only if it is an adjoint space, when considered as a Banach space.*

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2. Preliminaries. In this paper, we shall always deal with a C^*-algebra with unit I. Let M be a C^*-algebra, φ a linear functional. If $\varphi(a^*a) \geqq 0$ for all $a \in M$, it is said to be positive. Any positive linear functional on a C^*-algebra M is bounded and satisfies Schwarz's inequality; that is,

$$|\varphi(a^*b)| \leqq \varphi(a^*a)^{1/2}\varphi(b^*b)^{1/2}$$

for all $a, b \in M$; see [8].

An AW^*-algebra [5] is a C^*-algebra satisfying the following conditions: (a) In the set of projections, any collection of orthogonal projections has a least upper bound. (b) Any maximal commutative self-adjoint subalgebra is generated by its projections.

The notion of a Stonean space was introduced by M. H. Stone [9] as follows: a compact space Ω is said to be Stonean if it has the property that the closure of any open set is open and closed. Moreover, he showed that this property is equivalent to the following property: a uniformly bounded, increasing directed set of real valued continuous functions on Ω has a continuous function as a least upper bound.

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