OSCILLATION CRITERIA FOR LINEAR DIFFERENTIAL SYSTEMS WITH COMPLEX COEFFICIENTS

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1. Introduction. The basic oscillation and comparison theorems of the Sturmian theory for a self-adjoint second order linear differential equation with real coefficients have been extended to self-adjoint differential systems with real coefficients through the work of various authors. In this connection the reader is referred to the works of Morse [4], Birkhoff and Hestenes [1] and Reid [5], [6; Part II] listed in the bibliography at the end of this paper, and also to references to other literature on the subject cited by these authors.

The results of the present paper center around oscillation criteria for a self-adjoint linear differential system with complex-valued coefficients as developed in §§ 2 and 3. As a self-adjoint system with complex coefficients involving complex-valued dependent functions $u_1(x), \cdots, u_n(x)$ is equivalent to a self-adjoint system with real coefficients involving real-valued dependent functions $y_1(x), \cdots, y_{2n}(x)$, one might feel that all worthwhile criteria for a system with complex coefficients would be immediate consequences of known criteria for systems with real coefficients. Such is not the case, however, as appears in the treatment of §§ 2 and 3. For those portions of the theory of systems with complex coefficients that parallel closely the theory of systems with real coefficients the treatment is limited to a concise statement of results. Here no attempt is made to discuss for self-adjoint systems with complex coefficients the analogues of the general comparison and separation theorems obtained by Morse [4; Chapter IV] for self-adjoint systems with real coefficients. Also, no attention is given to systems with complex coefficients that are direct generalizations of the accessory equations for a variational problem of Bolza type, although many of our results have direct extensions to such systems. Certain aspects of these topics will appear in a subsequent paper on a problem related to that herein discussed.

Section 4 of this paper is devoted to specific criteria of oscillation and non-oscillation for self-adjoint systems. There are given certain criteria that are direct generalizations of results of Wintner [12] for a single equation of the second order, and there is stated without proof a theorem on a necessary and sufficient condition for non-oscillation near infinity that extends a result of Sternberg [7]. There is established also a sufficient condition for oscillation near infinity that extends

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