THE PROBLEM OF CONTINUOUS PROGRAMS

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1. Introduction. In a discrete programming problem one selects a policy at specified times which governs the behavior of some process during the succeeding time intervals; the problem is to find that *program*, that is, sequence of choices of policy, which maximizes the value of some pre-assigned functional associated with the process. It is of interest to learn how the values of the functional behave when policy-making decisions are required more and more frequently.

As an example of a discrete programming problem, suppose an investor re-distributes a fixed capital investment among N related businesses once a week. The income rate Q_i^k of the kth business during the ith week $[t_i, t_{i+1})$ depends on the *income* $q_i=(q_i^1, \cdots, q_i^N)$ up to the beginning of the week, where q_i^k is the income of the kth business, on the *policy*, that is, distribution of capital for the week, and on the time t_i. Suppose further that the businesses are risky in that if one fails all fail, and that the probability $P_i(t_{i+1}-t_i)$, of failure during a given week, assuming the businesses exist at the beginning of the week, depends on the policy for the week and the time of year. Setting $Q_i=(Q_i^1, \cdots, Q_i^N)$ and letting p_i represent the probability of survival up to time t_i, it is clear that q_i and p_i satisfy difference equations, stated more explicitly in § 2,

$$q_{i+1}-q_i=Q_i(t_{i+1}-t_i) \tag{1.1}$$

$$p_{i+1}-p_i=-p_iP_i(t_{i+1}-t_i)\ , \tag{1.2}$$

in which the right-hand sides at times t_i depend on q_i, p_i and a policy, which we represent as a point of the set X of all possible distributions of capital. The investor's programming problem is to select a policy for every week of the year which will maximize the expected total income

$$f=\sum_i p_i||Q_i||(t_{i+1}-t_i) \tag{1.3}$$

of all the businesses, where

$$||Q_i||=\sum_{k=1}^{N}|Q_i^k|\ . \tag{1.4}$$

It is assumed that he does not care what happens after the year is over.

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