CALCULATION OF AXIALLY SYMMETRIC CAVITIES AND JETS

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1. Introduction. We shall study in this paper steady, axially symmetric, irrotational flows of an incompressible liquid. We shall be interested in motions that exhibit a free surface along which the liquid is bounded by a gas whose inertia is negligible relative to that of the liquid, so that the gas can be assumed to be at rest and to have constant pressure. The determination of flows of this type is a mathematical problem of exceptional difficulty because the shape of the free surface is not known and must be calculated as part of the solution. In the case of axial symmetry, no systematic method has as yet been developed for finding free surface flows past prescribed obstacles, although a few calculations, notably those by Trefftz [17] and by Southwell and Vaisey [16], have been executed on a basis of inspired guesswork. The chief drawback of the work done by these investigators is that their successive approximations to the shape of the free surface are obtained by trial and error and are slow to converge.

Our hope in the present article is to present techniques for the systematic calculation of free surface flows, with emphasis on the axially symmetric case. Recent advances [3, 4, 5, 6, 7, 18] in the mathematical theory of cavitational flow form the basis for our method. Although rigorous proof of the convergence of the series expansion and of the iterative scheme which we shall use appears to be too difficult to undertake at this time, nevertheless sound theoretical reasons are given for expecting the procedures to converge. This is in contrast with other schemes that the author has seen suggested, such as the interation process for the Trefftz integral equation, which theory would predict to diverge for more or less the same reason that the classical Neumann series for solution of the Dirichlet problem diverges, namely, because the lowest relevant eigenvalue does not exceed 1. The difference here between our approach and the earlier ones is analogous to the difference between solving an equation $x=f(x)$ by Newton's method, for which we expect rapid convergence, and solving the same equation by successive approximations of the type $x_{n+1}=f(x_n)$, which will diverge if the derivative of the function $f(x)$ exceeds 1. The significant disagreement that will be found between our numerical results and the earlier work in the field can be attributed partly to this failure of convergence and partly to the insensitive nature of the guesswork involved in the other methods.

Received December 6, 1955.