SOME PROPERTIES OF DISTRIBUTIONS ON LIE GROUPS

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1. Introduction. Let G be a separable Lie group and let V be a complete, metrizable, topological vector space. The underlying space of G is a separable real analytic manifold so that we can define, by the methods of L. Schwartz (see [7], [12], [13]), the spaces $\mathscr{E}(V)$ of indefinitely differentiable maps of G into V, and $\mathscr{D}(V)$ which consists of those maps in $\mathscr{E}(V)$ which are of compact carrier. Their duals are $\mathscr{D}'(V)$, the space of distributions on G with values in V' (the dual of V), and $\mathscr{E}'(V)$ which is the space of distributions of compact carrier with values in V'.

By using the group structure in G, we can define the convolution $S*f \in \mathscr{E}(C)$ for any $S \in \mathscr{D}'(V)$, $f \in \mathscr{D}(V)$, where C is the complex plane. The main result of this paper is: Let $S \in \mathscr{D}'(V)$ have the property that $S*f \in \mathscr{D}(C)$ whenever $f \in \mathscr{D}(V)$; then $S \in \mathscr{E}'(V)$. Moreover, the topology of $\mathscr{E}'(V)$ is that obtained by considering each $S \in \mathscr{E}'(V)$ as defining the continuous linear transformation $f \to S*f$ of $\mathscr{D}(V) \to \mathscr{D}(C)$ and then giving this set of transformations the compact-open topology (see [6]). This generalizes the result of [6] in case G is a vector group and $V=C$.

This result is generalized to double coset spaces $L\backslash G/K$ where L and K are compact subgroups of G. In this form, the result will be used by the author and F. I. Mautner to generalize the Paley-Wiener theorem and the theory of mean-periodic functions of Schwartz (see [8]).

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2. Distributions on G. Instead of using the usual method of defining distributions on G, as for example in de Rham and Kodaira [12], we shall follow another approach which is more akin to the author's thesis [5]. We shall show that the two methods are equivalent.

By "function" we shall mean "complex-valued function" unless the contrary is specifically stated. "Linear" will mean "linear over the complex numbers" always. By 1 we denote the identity in G, and by $\mathfrak{g}$ we denote the Lie algebra of G. For any $Y \in \mathfrak{g}$, we denote by $t \to \exp(tY)$ the unique one parameter subgroup in G whose direction

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