ON MAPPINGS FROM THE FAMILY OF WELL ORDERED SUBSETS OF A SET

SEYMOUR GINSBURG

A simply ordered set E is called a *k-set* if there exists a simply ordered extension of the family of nonempty well ordered subsets of E, ordered by initial segments, into E. If E is not a k-set then it is called a *k'-set*. Kurepa [1;2] first discussed these sets. He showed that if E is a subset of the reals and if the smallest ordinal number α such that E does not contain a subset of order type α is ω_1, then E is a k'-set. In particular the rationals and the reals, denoted by R and R^+ respectively, are both k'-sets. In this paper the existence of k-sets and k'-sets is discussed further. Theorem 7 states that each simply ordered set E is a terminal segment of some k-set $F(E)$. It is not true, however, that each simply ordered set E is similar to an initial section of some k-set $F(E)$ (Theorem 2). Finally, in Theorem 10 it is shown that each infinite simply ordered group is a k'-set.

Following the symbolism in [1 ; 2] let E be a simply ordered set and ωE the family of all nonempty well ordered subsets of E, partially ordered as follows: For A and B in ωE, $A <_k B$ if and only if A is a proper initial segment of B.[1]

Definition. A function f from ωE to E is called a *k-function* on E, if $A <_k B$ implies that $f(A) < f(B)$.

If there exists a k-function on E, that is, from ωE to E, then E is called a *k-set*. If not, then E is called a *k'-set*.

THEOREM 1. *If f is a k-function on E, then for each nonempty well ordered subset W of E, there exists an element x in W such that $f(W) \leqq x$.*

Proof. Suppose that the theorem is false, that is, suppose that there exists an element W_1 in ωE with the property that $x < f(W_1)$ for each x in W_1. Let $W_2 = W_1 \cup f(W_1)$. It is easily seen that W_2 is well ordered, $W_1 <_k W_2$, $x < f(W_2)$ for each element x in W_2, and the order type of W_2 is $\geqq 2$. Suppose that for each $0 < \xi < \alpha$, W_ξ is an element

Received October 17, 1955. Presented to the American Mathematical Society November, 1955.

[1] A is a (proper) initial segment of B if A is a (proper) subset of B and if, for each element z in A, $\{x | x \leqq z, x \in B\}$ is a subset of A. A is a terminal segment of B if A is a subset of B and if, for each element z in A, $\{x | z \leqq x, x \in B\}$ is a subset of A.