CERTAIN GENERALIZED HYPERGEOMETRIC IDENTITIES OF THE ROGERS-RAMANUJAN TYPE

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1. Introduction. In a recent paper H. L. Alder [1] has obtained a generalization of the well-known Rogers-Ramanujan identities. In this paper I have deduced the above generalizations as simple limiting cases of a general transformation in the theory of hypergeometric series given by Sears [5]. This method, besides being much simpler than that of Alder, also gives a simple form for the polynomials $G_{k,\mu}(x)$ given by him. In Alder's proof the polynomials $G_{k,\mu}(x)$ had to be calculated for every fixed k with the help of certain difference equations but in the present case we get directly the general form of these polynomials.

2. Notation. I have used the following notation throughout the paper. Assuming $|x|<1$, let

$$(a)_s \equiv (a;\, s)=(1-a)(1-ax)\cdots(1-ax^{s-1}), \qquad (a)_0=1$$

$$\prod^{s}(a_1, a_2, \cdots, a_r; b_1, b_2, \cdots, b_t)=\frac{(a_1;\, s)(a_2;\, s)\cdots(a_r;\, s)}{(b_1;\, s)(b_2;\, s)\cdots(b_t;\, s)}$$

$$\prod(a)=\prod_{n=0}^{\infty}(1-ax^n)$$

$$\prod(a_1, a_2, \cdots, a_r; b_1, b_2, \cdots, b_t)=\frac{\prod(a_1)\prod(a_2)\cdots\prod(a_r)}{\prod(b_1)\prod(b_2)\cdots\prod(b_t)}$$

$$K_s=\frac{(k;\, s)(x\sqrt{k};\, s)(-x\sqrt{k};\, s)}{(x;\, s)(\sqrt{k};\, s)(-\sqrt{k};\, s)}$$

$$K_{s,r}=K_s\frac{(x^{-r};\, s)}{(kx^{r+1};\, s)}x^{rs}$$

$$S_{n,n-1}=\sum_{r_n=0}^{r_{n-1}}\frac{k^{r_n}x^{r_n^2}(x^{r_{n-1}-r_n+1};\, r_n)}{(x;\, r_n)}, \qquad S_{1,0}=\sum_{r_1=0}^{r}\frac{k^{r_1}x^{r_1^2}(x^{r-r_1+1};\, r_1)}{(x;\, r_1)}$$

$$T_{n,M}=\sum_{t_n=0}^{\left[\frac{M-n-1}{M-n}-t_{n-1}\right]}\frac{(x^{t_{n-1}-2t_n+1};\, 2t_n)x^{-2t_n(t_{n-1}-t_n)}}{(x;\, t_n)(x^{t_{n-2}-2t_{n-1}+1};\, t_n)}, \qquad (M=3, 4, 5, \cdots)$$

where $[a]$ denotes the integral part of a.

The numbers $s, r, r_1, r_2, \cdots, t, t_1, t_2, \cdots$ are either zero or positive

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