PERTURBATION OF THE CONTINUOUS SPECTRUM AND UNITARY EQUIVALENCE

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1. Introduction. Suppose that A and B are self-adjoint operators in a Hilbert space H such that $B-A=P$ is a completely continuous operator. We shall concern ourselves with the problem of finding conditions sufficient to guarantee that B is unitarily equivalent to A. Clearly a necessary condition is that the spectrum of A (considered as a point set on the real line) is equal to the spectrum of B. However this condition is not sufficient; von Neumann [8] has proved the following result

1.1. *Let A and C be bounded self-adjoint operators in a separable Hilbert space, such that the spectra of A and C have the same limit points. Then there exists an operator B that is unitarily equivalent to C and such that $B-A$ is completely continuous.*

Thus we see that perturbation by a completely continuous operator can radically alter the multiplicity of the spectrum. Even if A and B have pure continuous spectra on the same interval, it does not follow that B is unitarily equivalent to A.

Our present investigation continues along lines begun by Friedrichs in [1] and [2]. He considered bounded operators A that have continuous spectrum of finite multiplicity, and worked in the representation space where A corresponds to a multiplication operator. One of Friedrichs' results is the following.

1.2. *Let $H=L^2(-1, 1)$ and let A be the operator that sends any function $f(x)$ of H into $xf(x)$.. Let P be the integral operator with the hermitian kernel $p(x, y)=\overline{p(y, x)}$, where p satisfies certain Lipschitz conditions. Then if ε is a sufficiently small real number, there exist unitary operators U_ε and V_ε such that*

(i) $e^{-i(A+\varepsilon P)t}e^{iAt}$ *converges strongly to* U_ε as $t \to \infty$;

(ii) $e^{-i(A+\varepsilon P)t}e^{iAt}$ *converges strongly to* V_ε as $t \to -\infty$;

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