DISTRIBUTIVITY IN BOOLEAN ALGEBRAS

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1. Introduction. Let α be an infinite cardinal number; suppose B is an α-complete Boolean algebra, that is, every subset of B which contains no more than α elements has a least upper bound in B.

DEFINITION 1.1. B is *α-distributive* if the following identity[1] holds in B whenever S and T are index sets of cardinality $\leq \alpha$:

$$(1) \qquad \bigwedge_{\sigma \in S}(\bigvee_{\tau \in T} a_{\sigma\tau}) = \bigvee_{\varphi \in F}(\bigwedge_{\sigma \in S} a_{\sigma\varphi(\sigma)}), \qquad \text{where} \quad F = T^S .$$

This paper studies α-distributive Boolean algebras, their Boolean spaces and the continuous functions on these Boolean spaces. A survey of the main results can be obtained by reading Theorems 6.5, 7.1, 8.1 and 8.2.

2. Notation. Throughout the paper, α denotes a fixed infinite cardinal number. The term α-B.A. is used to abbreviate α-complete Boolean algebra. Only α-complete algebras are considered, although some of the definitions apply to arbitrary Boolean algebras. We speak of α-subalgebras, α-ideals, α-homomorphisms, α-fields, etc., meaning that the relevant operations enjoy closure up to the power α. For example, an α-field is a field of sets, closed under α-unions, that is, unions of α or fewer element.

The lattice operations of join, meet and complement are designated by $\vee$, $\wedge$ and $(')$ respectively. The symbols 0 and u stand for the zero and unit elements of a Boolean algebra. Set operations are represented by rounded symbols: $\cup$, $\cap$ and $\subseteq$ respectively denote union, intersection and inclusion. If A and B are sets, $B-A$ is the set of elements of B which are not in A; the complement (in a fixed set) of A is designated A^c. The empty set is denoted by 0. The symbol $\overline{\overline{A}}$ stands for the cardinality of the set A. Finally, for typographical reasons, we use the symbols 2^α and $\exp(\alpha)$ interchangeably.

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[1] The notion of α-distributibity was introduced in [**1**]. It is assumed that the least upper bound an the right side of the equality (1) exists. However, by Corollary 3.4 below, it would suffice to make the equality in (1) contingent on the existence of this least upper bound.