SIMPLIFIED PROOFS OF "SOME TAUBERIAN THEOREMS" OF JAKIMOVSKI

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1. Introduction. In this note, the preceding paper (mentioned in the title) will be referred to as [J], the papers or books numbered 1, 2, $\cdots$ in the bibliography concluding [J] will be referred to as [J1], [J2], $\cdots$, while those in the numbered list of references at the end will be referred to by their numbers in square brackets.

The notation in [J] is retained with a slight simplification as follows. As in Hardy's *Divergent series* [J3], a sequence $\{t_n\}$ is called a Hausdorff transform of another sequence $\{s_n\}$ when there is a sequence $\{\mu_n\}$ such that

$$(1) \qquad \Delta^n t_0 = \mu_n \Delta^n s_0 \,.$$

If α is a real number, the special case of $\{t_n\}$ defined by (1) with $\mu_n = (n+1)^{-\alpha}$, called the (H, α) transform, will be denoted by $H^\alpha s$ where s denotes the sequence $\{s_n\}$. Since two Hausdorff transformations are commutable, the operator H^α is such that $H^\alpha H^\beta = H^\beta H^\alpha = H^{\alpha+\beta}$ and H^0 is the identity operator.

From the Abel or (A) transform of $\{s_n\}$, defined as the left-hand member of

$$(2) \qquad (1-x)\sum_0^\infty s_n x^n = (-1)^p (1-x)^{-p+1} \sum_0^\infty \Delta^p s_{n-p} x^n \,, \qquad 0 < x < 1,\ p = 1, 2, 3, \cdots,$$

we deduce the equality (2) by induction on p. It is in the form of the right-hand member of (2) that the (A) transform is used in this note.

For any sequence $\{s_n\}$, summability (H, α) to a finite value l and summability (A) to l have their usual meanings as in [J].

2. The fundamental theorem in [J]. This theorem ([J], Theorem 2) may be restated as follows with its non-trivial parts separated, so that Tauber's first theorem ([J3], Theorem 85) emerges as the case $k=1$ of the first part, with the conclusion of the convergence of $\{s_n\}$ restated as that of the (H, -1) summability of $\{s_n\}$.

THEOREM 1. (a) *If* (i) $\{s_n\}$ *is summable* (A) *to* l, (ii) *for a positive integer* k, $n^k \Delta^k s_{n-k} = o(1)$, $n \to \infty$, *then* $\{s_n\}$ *is summable* (H, $-k$) *to* l.

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