SOME TAUBERIAN THEOREMS

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1. Introduction. The following Taurberian theorem is well known.

THEOREM A. *If the sequence* $\{s_n\}$, $n=0, 1, 2, \cdots$, *is summable Abel*[1] *to* s *and the sequence* $\{n(s_n-s_{n-1})\}$ *is bounded on one side, then* $\{s_n\}$ *is convergent to* s.

Another Tauberian theorem, proved in [4], is

THEOREM B. *If the series* $\sum_{n=0}^{\infty} a_n$ *is summable Abel to* s *and the sequence* $\{n^2(a_{n-1}-a_n)\}$ *is bounded on one side, then* $\lim_{n\to\infty} na_n=0$.

An immediate consequence of Theorem B is the well known proposition that, for a convergent series $\sum_{n=0}^{\infty} a_n$ with monotonically decreasing terms, $\lim_{n\to\infty} na_n=0$.

By a well known theorem of Tauber, the series $\sum_{n=0}^{\infty} a_n$ of Theorem B is convergent and hence the sequence $\{s_n\}$ of partial sums of the series is summable $(H, -1)$, that is, $\{s_n\}$ is summable by the Hölder method of order -1, as defined in § 2. Thus Theorem B is equivalent to the following

THEOREM C. *If the sequence* $\{s_n\}$, $n=0, 1, 2, \cdots$, *is summable Abel to* s *and the sequence* $\left\{\binom{n}{2}(s_{n-2}-2s_{n-1}+s_n)\right\}$ *is bounded on one side, then* $\{s_n\}$ *is summable by the Hölder method of summability of order* -1.

As will be shown below both Theorem A and Theorem C are special cases of general results proved in § 5 of this paper.

The Tauberian conditions,

$$\binom{n}{1}(s_{n-1}-s_n)=O_L(1)$$

and

Received September 16, 1955 and in revised forms January 16, 1956, and May 28, 1956.

[1] Concepts and propositions mentioned or used in this paper without definition or proof are to be found in Hardy's book [3].