STRUCTURE THEORY FOR A CLASS OF CONVOLUTION ALGEBRAS

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Introduction. This paper is a chapter in the study of convolution algebras begun in [7]. The algebras studied here are algebras of Borel measures on certain compact semigroups, and we describe completely the structure of these algebras. The solution obtained seems remarkable in view of the extreme complexity of the corresponding measure algebras for compact Abelian groups (see [12]). Our success is explained by the simple algebraic structure of the semigroups we deal with.

In addition to the structure theory (§§ 2–6), we give an application to probability (§ 7), and some concrete examples and illustrations (§ 8).

Throughout this paper, we use the notation and terminology of [7]. In particular, the reader should be familiar with § 1 of [7]. The related papers [6] and [8] are not essential for understanding the present paper, but are referred to occasionally here at points of contact in subject-matter. For all measure-theoretic terms and techniques not explained here, see [4]. References are made throughout the present paper to [9] for topological matters, and to [10] for the elementary theory of Banach algebras. We use K to denote the complex number system. All other special symbols will be explained as they appear.

1. The semigroups to be studied.

1.1. We consider an arbitrary non-void set G, completely ordered by a transitive, irreflexive relation "$<$". That is, for all $x,\ y \in G$, exactly one of the relations $x<y$, $x=y$, $y<x$ obtains, and the relations $x<y$ and $y<z$ imply $x<z$. As usual, we write $y>x$, meaning $x<y$, and we write $x \leqq y$, meaning $x<y$ or $x=y$. For $u, v \in G$, we define

$$]u,\ v[=\{x\colon x\in G,\ u<x<v\} \qquad \text{(open interval)}\ ,$$

$$[u,\ v[=\{x\colon x\in G,\ u\leqq x<v\} \qquad \text{(half-open interval)}\ ,$$

$$]u,\ v]=\{x\colon x\in G,\ u<x\leqq v\} \qquad \text{(half-open interval)}\ ,$$

$$[u,\ v]=\{x\colon x\in G,\ u\leqq x\leqq v\} \qquad \text{(closed interval)}\ .$$

These sets may or may not be void, depending upon the relation between u and v.

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