THE NUMBER OF DISSIMILAR SUPERGRAPHS OF A LINEAR GRAPH

FRANK HARARY

1. Introduction. A (p, q) *graph* is one with p vertices and q lines. A formula is obtained for the number of dissimilar occurrences of a given (α, β) graph H as a subgraph of all (p, q) graphs G, $\alpha \leqq p$, $\beta \leqq q$, that is, for the number of dissimilar (p, q) supergraphs of H. The enumeration methods are those of Pólya [7]. This result is then applied to obtain formulas for the number of dissimilar complete subgraphs (cliques) and cycles among all (p, q) graphs. The formula for the number of rooted graphs in [2] is a special case of the number of dissimilar cliques. This note complements [3] in which the number of dissimilar (p, k) subgraphs of a given (p, q) graph is found. We conclude with a discussion of two unsolved problems.

A (*linear*) *graph* G (see [5] as a general reference) consists of a finite set V of *vertices* together with a prescribed subset W of the collection of all unordered pairs of distinct vertices. The members of W are called *lines* and two vertices v_1, v_2 are *adjacent* if $\{v_1, v_2\} \in W$, that is, if there is a line joining them. By the *complement* G' of a graph G, we mean the graph whose vertex-set coincides with that of G, in which two vertices are adjacent if and only if they are not adjacent in G.

Two graphs are *isomorphic* if there is a one-to-one adjacency-preserving correspondence between their vertex sets. An *automorphism* of G is an isomorphism of G with itself. The *group of a graph* G, written $\Gamma_0(G)$, is the group of all automorphisms of G. A *subgraph* G_1 of G is given by subsets $V_1 \subseteqq V$ and $W_1 \subseteqq W$ which in turn form a graph. If H is a subgraph of G, we also say G is a *supergraph* of H. Two subgraphs H_1, H_2 of G are *similar* if there is an automorphism of G which maps H_1 onto H_2. Obviously similarity is an equivalence relation and by the number of *dissimilar* vertices, lines, $\cdots$ of G, we mean the number of similarity classes (as in [3, 4, 6]).

Two supergraphs G_1 and G_2 of H are *H-similar* if there exists an isomorphism between G_1 and G_2 which leaves H invariant. It is clear that the number of dissimilar (p, q) supergraphs of H is equal to the number of dissimilar occurrences of H as a subgraph of all (p, q) graphs.

2. Supergraphs. Let H be an arbitrary (α, β) graph. We wish to enumerate the dissimilar (p, q) supergraphs of H where $p \geqq \alpha$, $q \geqq \beta$.

Received March 2, 1956. Presented to the American Mathematical Society August 31, 1955. Written while the author was a member of the Technical Staff of the Bell Telephone Laboratories, Murray Hill, New Jersey, Summer 1955.