ON CHARACTERISTIC FUNCTIONS OF BANACH SPACE VALUED RANDOM VARIABLES

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1. Introduction. In recent years several authors have considered the notion of random variables with values in a Banach space, $\mathfrak{X}$. One of the basic problems is to characterize those positive definite functions on $\mathfrak{X}^*$ that are characteristic functions of such random variables. Mourier [4] has given a solution to this problem if $\mathfrak{X}$ is separable and reflexive. The purpose of this paper is to give another solution of this problem. Our results are valid if $\mathfrak{X}$ is reflexive. However the contribution of this paper is not so much the removal of the condition of separability, rather we feel that our method sheds new light on the problem and aids in understanding it. The basic tool that we use is the concept of a weak distribution as introduced by Segal [5], and this idea succeeds in unifying the theory.

Section 2 contains the basic definitions and preliminaries. The main results are contained in § 3 but in a form slightly more general than needed for the problem at hand. However we will need the results in this generality in a future paper. The contents of § 3 are clearly valid in any locally convex linear topological space. Finally in § 4 our solution to the problem stated above is given along with some examples and consequences.

The considerations of Bochner in chapters five and six of [1] are somewhat related to our problem.

2. Definitions. Let $(\Omega, \mathfrak{F}, P)$ be a probability space, that is, Ω is an abstract point set, $\mathfrak{F}$ a σ-algebra of subsets of Ω, and P is a measure on $(\Omega, \mathfrak{F})$ with $P(\Omega)=1$. Let $\mathfrak{X}$ be a real Banach space[1] and $\mathfrak{X}^*$ its conjugate space. Let $X: \Omega \to \mathfrak{X}$, we will call X an $\mathfrak{X}$ *valued random variable* if X is weakly measurable, that is, if $\langle x^*, X(\omega)\rangle$ is a real valued $\mathfrak{F}$-measurable function for each $x^* \in \mathfrak{X}^*$. Let $E(X)$ be the Pettis integral of X with respect to P, provided it exists. Thus $E(X)$ is the unique element of $\mathfrak{X}$ such that $\langle x^*, E(X)\rangle = E\{\langle x^*, X\rangle\} = \int \langle x^*, X(\omega)\rangle dP$ for each $x^* \in \mathfrak{X}^*$. The *characteristic function* of X is defined as follows,

$$\phi(x^*) = E\{e^{i\langle x^*, X\rangle}\} = \int e^{i\lambda} dF((x^*; \lambda) \tag{2.1}$$

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[1] The extension to a complex Banach space is essentially clear.