REGULAR REGIONS FOR THE HEAT EQUATION

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1. Introduction. Let R be a region (open connected set) in the plane or in space ($x=[x_1, x_2]$ or $x=[x_1, x_2, x_3]$). We will say that R is a regular region for Laplace's equation

$$\Delta u=0 \tag{1}$$

if the Dirichlet problem for R always has a solution for continuous data. By this we mean: given a function $\psi(\xi)\in C$ (that is, continuous) for $\xi\in B$, the boundary of R, there is a unique function $u(x)\in C$ for $x\in\overline{R}=R\cup B$, for which

$$\Delta u=0 \qquad x\in R\,,$$

$$u(\xi)=\psi(\xi) \qquad \xi\in B\,.$$

We will further say that R is regular for the heat equation

$$\Delta u=u_t \tag{2}$$

if the "Dirichlet problem" for the heat equation has a solution for continuous data, that is, if for each

$$\phi(x)\in C \qquad x\in\overline{R}$$

and

$$\psi(\xi,\ t)\in C \qquad \xi\in B,\ t\geqq 0$$

where

$$\phi(\xi)=\psi(\xi,\ 0)$$

there is a unique function $u(x, t)\in C$, for $x\in\overline{R}$, $t\geqq 0$ for which

$$\Delta u=u_t \qquad x\in R,\ t>0$$

$$u(x,\ 0)=\phi(x) \qquad x\in\overline{R}$$

$$u(\xi,\ t)=\psi(\xi,\ t) \qquad \xi\in B,\ t\geqq 0\,.$$

Tychonoff [4] has shown that if R is bounded and regular for

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