HOLOMORPHIC FUNCTIONALS AND COMPLEX CONVEXITY IN BANACH SPACES

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1. Introduction. The present paper extends some basic theorems of the theory of several complex variables to Banach spaces. Results which are new even for finite dimension are also obtained. Considerable use is made of methods developed in "Complex Convexity" (Bremermann [8]), however, many modifications are necessary to adapt them to infinite dimension.

A complex valued functional is *Gâteaux holomorphic* (or in short *G-holomorphic*) in a domain D of a complex Banach space B_c if it is single valued and its restriction to an arbitrary analytic plane $\{z|z=z_0+\lambda a\}$ ($z_0 \in D$, $a \in B_c$, λ a complex parameter) is a holomorphic function of λ in the intersection of the plane with D. The space of n complex variables C^n can be considered as a Banach space, and for C^n the above definition is equivalent to the usual definition of a holomorphic function of several complex variables. In an infinite dimensional Banach space the Gâteaux holomorphic functions are not necessarily locally bounded, while in a finite dimensional space the local boundedness is a consequence of holomorphy. Therefore another notion of holomorphy, also coinciding with the notion of holomorphy in finite dimensional spaces, is possible: A function is *Fréchet holomorphic* in a domain D if it is Gâteaux holomorphic and locally bounded (compare Hille [11] and Soeder [17]). The theories of both types of holomorphic functions have been studied, the latter more than the former. Both theories are considerably less developed than the theory of finitely many variables. This may be partly due to the fact that the infinite dimensional spaces are not locally compact, in fact, if a space is locally compact, then it is finite dimensional (see Hille [11]).

In the present paper the theory of Gâteaux holomorphic functionals is studied exclusively. As a tool are used plurisubharmonic functionals (as defined by Oka [14] and [15], Lelong [12] and Thorin [19]) and a functional $d_D^{(N)}(z)$ which is the distance of the point z from the boundary of the domain D measured in the norm N. A notion of holomorphic continuation is defined and a "basic lemma" on the simultaneous continuation of G-holomorphic functionals is proved (3.1).[1] A consequence of

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[1] The lemma in its present form is new also for finite dimension and permits to construct the envelope of an arbitrary domain in the C^n explicitely. This will be carried out in a further paper.