ZERO-DIMENSIONAL COMPACT GROUPS OF HOMEOMORPHISMS

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1. Introduction. All spaces and topological groups referred to in this paper will be compact and metric. All topological groups will additionally be zero-dimensional, that is, either finite or homeomorphic to a Cantor set. As general references we cite Zippin [6] and Montgomery and Zippin [4]. Several of our definitions are similar to those in [6].

A *topological transformation group* of a topological space is an association of a topological group G and a topological space E in the sense that each element g of G and point x of E determine a unique point of E. If this point be called x', we write $gx=x'$. The association is subject to the following conditions:

(1) if e denotes the identity of G, $ex=x$ for all $x \in E$,

(2) $g(g'x)=(gg')x$, $g, g' \in G$, $x \in E$, and

(3) gx is continuous simultaneously in g and x.

Each element of G may, under the association, be regarded as a homeomorphism of E onto itself.

The topological transformation group G is said to be *effective* if for each $g \in G$ not the identity, there is an $x_g \in E$ for which $gx_g \neq x_g$ and is said to be *strongly effective* (or *fixed-point-free*) if for each $g \in G$ not the identity and for each $x \in E$, $gx \neq x$. We shall use the symbol $Tg(G, E)$ to denote a particular association of G with E such that G is an effective topological transformation group of E. Thus by $Tg(G, E)$ we mean a particular group of homeomorphisms of E onto itself, the group being isomorphic to and identified with G. If $Tg(G, E)$ is strongly effective we write $TgS(G, E)$.

For $x \in E$, $G(x)$ will denote the set of all images of x under G and will be called the orbit of x under G. Similarly for $X \subset E$, $G(X)$ will denote the set of images of X under G. The individual orbits may be regarded as the "points" of a space, the orbit space, $O[Tg(G, E)]$ of $Tg(G, E)$. $O[Tg(G, E)]$ is a continuous decomposition of E.

The main purpose of this paper is to prove the following theorems:

THEOREM 1. *Let G be any compact zero-dimensional topological group. Let M be the universal curve.*[1] *Then there exists a $TgS(G, M)$*

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[1] The universal curve is a particular one-dimensional locally connected continuum. Its description and a characterization of it are given in § 3.