DIMENSION AND NON-DENSITY PRESERVATION OF MAPPINGS

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1. Introduction. In this paper consideration is given to conditions under which the property of being non-dense in a space in the sense of containing no open set in that space is invariant under certain types of mappings. In some spaces and for some mapping types the issue involved is essentially equivalent to the question of dimensionality preservation. These questions are of interest and importance in numerous mathematical fields. They are especially so in the study of topological aspects of the theory of functions and it is toward this connection that the results and methods in this note will be largely directed.

A single valued continuous transformation $f(X)=Y$ will be called a *mapping*. Such a mapping is *open* if open sets in X have open images in Y and is *light* provided $f^{-1}(y)$ is totally disconnected for each $y \in Y$. Also f has *scattered point inverses* provided that for each $y \in Y$, $f^{-1}(y)$ is a *scattered set* in the sense that no point of $f^{-1}(y)$ is a limit point of $f^{-1}(y)$.

As indicated above, a set K in a space X is *non-dense in* X provided K contains no open set in X. On the other hand that K is *dense in* X means that every point of X is either a point or a limit point of K. A mapping $f(X)=Y$ is said to *preserve non-density* for compact sets provided that $f(K)$ is non-dense in Y whenever K is compact and non-dense in X. For a mapping $f(X)=Y$, a subset X_0 of X is said to be *semi-dense* in X provided X_0 is dense in some open subset of every open set U in X whose image $f(U)$ is also open in Y. Thus the property of semi-density is a property of a subset of X *relative to a mapping* f *on* X and not an intrinsic property of X_0 alone.

For a mapping $f(X)=Y$, the set of all $x \in X$ such that x is a component of $f^{-1}f(x)$ will be designated by the symbol D_f. Also the symbol L_f will be used for the set of all $x \in X$ such that $f^{-1}f(x)$ is totally disconnected. Thus L_f is the maximum inverse set in X on which the mapping f is light, where by an *inverse set* I we mean a set which is the inverse of its transform under f, that is, one satisfying the relation

$$I=f^{-1}f(I) .$$

Accordingly L_f may be thought of as the *lightness kernel* or *0-dimensional kernel* of the mapping f. Obviously we have $L_f \subset D_f$.

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