LIE ALGEBRAS OF LOCALLY COMPACT GROUPS

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1. Introduction. We call an LP-group, a group which is the projective limit of Lie groups. Yamabe [8] has proved that every connected locally compact group is an LP-group. This permits the extension to locally compact groups of the notion of a Lie algebra. In §§ 2 and 3 we prove the existence and uniqueness of the Lie algebra of an LP-group and show the connection of the Lie algebra with the group by means of the exponential mapping.

In § 4, we extend the notion of a universal covering group for connected groups with the same Lie algebra. A covering group of a connected group g, in the extended sense used here, means a pair $(\bar{g}, w)$, where $\bar{g}$ is a connected LP-group and w is a continuous representation of $\bar{g}$ into g which induces an isomorphism of the Lie algebra of $\bar{g}$ onto the Lie algebra of g (see Definition 4.5). The universal covering group of a connected locally compact group is not necessarily locally compact and may not map onto the group. It turns out that the arc component of the identity in $\bar{g}$ is a covering space in the sense of Novosad [5] of the arc component of the identity of g (these components are dense subgroups, Lemma 3.7).

Finally, in § 5, we establish a one-to-one correspondence between "canonical LP-subgroups" of a group and subalgebras of its Lie algebra.

2. Projective limit of Lie algebras.

DEFINITION 2.1. By a *topological Lie algebra* (over the real numbers) we shall mean a (not necessarily finite dimensional) Lie algebra with an underlying topology such that the operations of addition, multiplication and scalar multiplication are continuous.

DEFINITION 2.2. Let J be an inductive set. Suppose given for each $a \in J$, a topological Lie algebra G_a such that if $a < b$ there exists a continuous representation $f_{ab}: G_b \rightarrow G_a$. Let $G = [\{X_a\} \in \prod_{a \in J} G_a$ such that $f_{ab} X_b = X_a$, all $a, b \in J$ with $a < b]$. Then G is a closed topological subalgebra of the direct product.

In analogy to A. Weil [7, p. 23], G will be called the *projective limit* of the G_a ($G = \lim G_a$) if the following hold:

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