

ON A NEW RECIPROCITY, DISTRIBUTION AND DUALITY LAW

G. KUREPA

Introduction. One knows various operations on sets, e. g. join, intersection, limit, A -operation (Suslin), etc. In the present article we define, as an extension of operations we introduced in another paper (Kurepa [6], [7]) several operations of considerable generality and importance. It turns out that the well-known *distribution law* (cf. § 11) as well as the *De Morgan duality principle* (cf. § 5) are very special cases of our theorems. Moreover, a new *reciprocity phenomenon* occurs (cf. § 12). All depend on the interconnection between *maximal chains* and *maximal antichains* of ordered sets. By considering ordered sets one achieves considerable generality. By their use we get a *synthetic view* on (1) the *analytic operation*; (2) *c-analytic operation* (definition of complements of analytic sets); (3) the *distribution law*; (4) the *duality law*; and moreover, one arrives at (5) a *new reciprocity law*. In particular, in connection with the distributive law, the maximal chains and maximal antichains indicate respectively two distinct ways to reach the same result (cf. Theorems 4.2, 8.1). On the other hand, the parallel considerations of maximal chains and maximal antichains of S give rise to a new kind of interconnection of elements of P^1 (1 being any set; cf. the k -condition in § 8). This in turn opens a broad way to new investigations by consideration of the elements of $P^{\alpha 1}$ instead of those of P^1 . Our results may be interpreted in mathematical logic too.

The results of this paper are connected to an idea we expressed in our Thesis [4], 135 n°40 (cf. A. Tarski [11]).

GLOSSARY AND NOTATIONS

Antichain; an ordered set having no couple of distinct comparable points.

Chain; an ordered set having no two distinct incomparable points.

1 or U means universal set.

rT (cf. 10.1)

Disjunctive family; a family composed of pairwise disjoint sets.

ϵ' denotes "not ϵ ."

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