# A CONVERGENCE THEOREM FOR A CERTAIN CLASS OF MARKOFF PROCESSES

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**1. Introduction.** The object of this paper is to generalize, by means of an approach due to S. Karlin [9], a theorem originally obtained by Bellman, Harris and Shapiro [1] which may be stated in the following way:

A system is considered whose state may be described by a point $t$ in the interval [0, 1]. A probability measure $\mu$ is given for the initial state of the system. At the end of each unit interval of time, one of the transformations $A_0$, $A_1$ is applied to the state $t$ with probabilities $\phi_0(t)$, $\phi_1(t)$ respectively, where $\phi_0(t)+\phi_1(t)=1$. The transformations are defined by

$$A_0t=\lambda_0t\,,\quad A_1t=\lambda_1t+(1-\lambda_1)\,,\quad 0\leqq\lambda_0,\ \lambda_1<1^1. \tag{1.1}$$

The assumption is made that

$$\phi_0(t)=1-t\,,\quad \phi_1(t)=t\,. \tag{1.2}$$

It is clear that (1.1) and (1.2) ensure that the end-points of the interval [0, 1] are absorbing, that is, if the state of the system is either 0 or 1, it remains so. Let $T\mu$ be the probability measure at the end of the first unit interval. It is then proved that as $n\to\infty$, $T^n\mu$ (that is, the probability distribution for the state of the system at time $n$) converges in distribution to a distribution concentrated at the points 0, 1 and the form of this limiting distribution which depends on $\mu$ is obtained.

The motivation for the consideration of such a system arose from certain learning models introduced by Bush and Mosteller. These are described in detail in their recent book [2]. (Condition (1.2) means that the state of the system may be identified with the probability of applying $A_1$).

The methods used in [1] to obtain the convergence of $T^n\mu$ are probabilistic. Karlin [9] considers the space of continuous functions on the unit interval and obtains a bounded operator $U$ on this space whose adjoint is $T$. A convergence theorem is obtained for $U^n$ and the result is translated into the adjoint space (that is, the space of measures) to

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[1] Karlin also considers boundary cases where $\lambda_0$, $\lambda_1$ may be 1.