MONOTONE MAPPINGS OF MANIFOLDS

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1. Introduction. Mappings of the 2-sphere, and more generally of the 2-manifolds, have been studied by various authors. (See, for instance, [9] and references therein, [7].) Generally, these mappings have been subjected to certain "monotoneity" conditions on the counter-images of points. Thus, in Moore's first paper [8] on the 2-sphere, it was required not only that counter-images be connected, but that they not separate the sphere. In terms of homology, then, he required of a counter-image C that $p_a^r(C)=0$ for $r=0,1$. Later studies of Moore and others usually omitted the requirement that $p^1(C)=0$, thus increasing the possible number of topological types of images. With the condition $p^1(C)=0$ imposed, the image of the 2-sphere is a 2-sphere, and of a 2-manifold is a 2-manifold of the same type. Without this condition, the various types of "cactoids" are obtained.

In the present paper we consider some higher dimensional cases. As might be expected, we impose higher dimensional "monotoneity" conditions.

DEFINITION 1. A mapping f: $A \rightarrow B$ is called *n-monotone* if $H^r(f^{-1}(b))=0$ for all $b \in B$ and $r \leq n$. (See [10; p. 904].)

EXAMPLE. Let us consider the mapping induced by decomposing the 3-sphere into disjoint closed sets each of which is a point, except that all points on some suitable "wild" arc [5; Ex. 1.1] A are identified. This mapping is r-monotone for all r, but the image-space is no longer a 3-sphere; indeed, it is not a 3-manifold in the classical sense at all, since the point corresponding to A does not have a 3-cell neighborhood.

This example makes it at first appear that because of such "homotopy" difficulties, it may be useless to look for any well-defined class of configurations in higher dimensions. However, as we show below, the class of configurations obtained is precisely that of the generalized manifolds. Moreover, we need not restrict the mappings to the mappings of 3-manifolds in the classical sense, since the generalized manifolds turn out to form a class which is closed relative to the mappings considered. This result forms, then, a new justification for the study of generalized manifolds.

2. Preliminary theorems and lemmas. In general, spaces are

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