RETRACTIONS IN SEMIGROUPS

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Let S be a semigroup (that is, a Hausdorff space together with a continuous associative multiplication) and let E denote the set of idempotents of S. If $x \in S$ let

$$L_x = \{y | y \cup Sy = x \cup Sx\}$$

and

$$R_x = \{y | y \cup yS = x \cup xS\} .$$

Put $H_x = L_x \cap R_x$ and for $e \in E$ let

$$H = \cup \{H_e | e \in E\} ,$$

$$M_e = \{x | ex \in H \text{ and } xe \in H\} ,$$

$$Z_e = H_e \times (R_e \cap E) \times (L_e \cap E)$$

and

$$K_e = (L_e \cap E) \cdot H_e \cdot (R_e \cap E) .$$

Under the assumption that S is compact we shall prove that K_e is a retract of M_e and that K_e and Z_e are equivalent, both algebraically and topologically. This latter fact sharpens a result announced in [6] and the former settles several questions raised in [7].

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LEMMA 1. *Let $Z = S \times S \times S$ and define a multiplication in Z by*

$$(t, x, y) \cdot (t', x', y') = (txy't', x', y) ;$$

then Z is a semigroup and, with this multiplication, the function f: $Z \to S$ defined by $f(t, x, y) = ytx$ is a continuous homomorphism.

The proof of this is immediate. *We use only the above defined multiplication in Z and not coordinatewise multiplication.* It is clear that $f(Z_e) = K_e$.

Since the sets H_e, $e \in E$, are pairwise disjoint groups [1] it is legitimate to define functions

$$\eta : H \to E , \qquad \theta : H \to H$$

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