DEVELOPMENT OF THE MAPPING FUNCTION AT AN ANALYTIC CORNER

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1. Introduction. In this paper we shall apply some theorems proved in [3] to study the following problem in conformal mapping. Let D be a domain of the complex plane, the boundary of which in the neighborhood of the origin consists of portions of two analytic curves Γ_1 and Γ_2. Suppose Γ_1 and Γ_2 meet at the origin and form a corner with opening $\pi\alpha > 0$, and suppose the origin is a regular point of both curves. Let $F(z)$ be a function which maps conformally the upper half plane $\Im z > 0$ onto the domain D, and suppose that $F(0)=0$. How does the mapping function $F(z)$ behave in the neighborhood of the origin?

A partial answer to this question is given by a theorem stated by Lichtenstein [5]. Let $F^{-1}(z)$ be the inverse function which maps D onto the upper half plane. Then Lichtenstein stated that for z in the neighborhood of the origin

$$\frac{dF^{-1}(z)}{dz} = z^{1/\alpha - 1}\varphi(z)$$

where $\varphi(z)$ is a continuous function with $\varphi(0) \neq 0$.[1] This same result can, however, be obtained with much weaker requirements on the boundary curve as has been shown by the work of Kellogg [2] and Warschawski [6].

In the case $\alpha=1$ where the curves Γ_1 and Γ_2 meet at a straight angle Lewy [4] has proved a much stronger result—that $F(z)$ has an asymptotic expansion in powers of z and $\log z$. The method used in this paper is a generalization of that used by Lewy. We find that for all $\alpha > 0$ the function $F(z)$ has an asymptotic expansion in the neighborhood of the origin. If α is irrational then the expansion is in integral powers of z, and z^α. If α is rational then the expansion is in integral powers of z, z^α, and $\log z$.

2. Notation. First let us make clear what type of asymptotic expansions we will be considering. Let $\chi_n(z)$, $(n=0, 1, 2, \cdots)$ be a sequence of functions such that $\chi_{n+1}(z)/\chi_n(z) \to 0$ as $z \to 0$ in the sector

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[1] Lichtenstein proved this result only in the case of irrational α. The complete theorem has been proved recently by Warschawski [7].