PERTURBATION OF DIFFERENTIAL OPERATORS

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Introduction. N. Dunford, in a series of papers [3, 4, 5], has initiated the study of operators on Banach spaces that allow a representation analogous to the Jordan canonical form for operators on a finite dimensional vector space. Such operators he has called spectral operators. They include, of course, self-adjoint operators which have found such wide application to problems of analysis. J. Schwartz [9] has exhibited an interesting class of spectral operators which contains many classical ordinary differential operators. His chief tool was a perturbation theorem that guarantees that if T is a regular spectral operator with a discrete spectrum that converges to infinity sufficiently rapidly and B is a bounded operator, then $T+B$ is again a regular spectral operator. This result provides a tool for showing that second order differential operators with suitable boundary conditions are regular spectral but does not suffice for proving this property for differential operators of higher order. This paper refines the method of J. Schwartz to allow application also to differential operators of higher order by showing that under certain conditions a regular spectral operator T may be perturbed by an unbounded operator S with the result that $T+S$ is still regular spectral.

The paper is divided into three parts. The first part presents preliminary notions and lemmas to be used in part II where the principal theoretical tool is fashioned in Theorem 1. Its object is to set forth conditions under which an operators is spectral (see Definition 1). This problem is attacked in the following form. Suppose that T is known to be a spectral operator. Under what hypotheses on T and a perturbing operator S may it be said that the operator $T+S$ is spectral? An answer to this question is given in Theorem 1. This theorem is then applied in the third part to differential operators of even order with "separated" boundary conditions on a finite interval. First, the simple operator defined by means of the formal differential operator $\frac{d^{2\mu}}{dx^{2\mu}}$ and "separated" boundary conditions is shown to be spectral. Then, with the aid of Theorem 1, the perturbed operator

$$\frac{d^{2\mu}}{dx^{2\mu}}+\sum_{i=2}^{2\mu}Q_i\frac{d^{2\mu-i}}{dx^{2\mu-i}}$$

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