REPRESENTATION THEOREMS FOR CERTAIN FUNCTIONAL OPERATORS

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1. Introduction. Almost all the operators arising in applications of the Heaviside operational calculus share two properties. The precise formulation of these properties may vary, but their general nature is, in the first case, a commutativity rule relating to the operation of semi-translation, whilst in the second case it is a condition of continuity of some sort. Possible precise formulations of these conditions are typified by postulates (O_1), (O_2) and (O_2'), which appear subsequently. Verification of the opening remark is to be found by glancing at the diverse illustrations of the technique to be found for example throughout [4].

It is the aim of the present paper to base proofs of general representation theorems upon such characteristic properties. The appropriate theorems will depend of course on the topologies envisaged in the continuity condition. Because of this, neither theorem proved here applies to all conceivable "operational expressions": an outlaw expression would be $\exp(hp)(h>0)$, for instance. Modifications are possible, however, and would lead to theorems covering wider ranges of operational expressions.

As is well known, if the operands are restricted suitably, the operational calculus can be formulated in terms of the one-sided Laplace transform. Special attention is given to this case, and the corresponding representation theorem can be looked upon as a solution of the problem of factor functions for the Laplace transformation. The methods employed were suggested by those used in [3] to study factor functions for the Fourier transformation.

The general nature of all results obtained is very close to one given by L. Schwartz [5, p. 18, Théorème X].

2. Classes of functions and operators. The widest class of functions to be considered will be denoted by $\mathscr{F}$ and will consist of those functions $f=f(t)$ which are defined and locally integrable on the half-line $R_+=\{t:t>0\}$. Functions which are equal a.e. are identified. A fundamental operator mapping $\mathscr{F}$ into itself is "semi-translation by s", where $s\geqq 0$: this is denoted by U_s and is defined by

$$U_sf(t)=\begin{cases} f(t-s) & \text{for } t>s\,, \\ 0 & \text{for } 0<t\leqq s\,. \end{cases} \tag{2.1}$$

The first of the two characteristic properties to be postulated about

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