SIMPLIFIED PROOFS OF "SOME TAUBERIAN THEOREMS" OF JAKIMOVSKI: ADDENDUM AND CORRIGENDUM

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Dr. B. Kuttner has kindly drawn my attention to a paper by F. Hausdorff, *Die Äquivalenz der Hölderschen und Cesaròschen Grenzwerte negativer Ordnung*, Math. Z., **31** (1930), 186–196, which contains a generalization of Jakimovski's fundamental theorem discussed in § 2 of my paper (this volume, pp. 955–960) and Szász's product-theorem referred to in § 3 of my paper, under numbers VI and III respectively in the list of numbered results I–VIII. There is a close connection between Hausdorff's paper and mine, as shown, for instance, by a comparison of Lemmas 1, 3 in the latter with the interpretation of Γ_{-k} and the result numbered VII in the former (pp. 195–6). It is unfortunate that I should have been ignorant of Hausdorff's paper and that the paper should have escaped mention in the lists of references provided by such works as G. H. Hardy's *Divergent series* and O. Szász's *Introduction to the theory of divergent series.*

Dr. Kuttner has also been good enough to call my attention to the fact that my step numbered (6) in p. 958 is not a valid deduction from my Lemma 2. For the convenience of the reader, I add that my incorrect argument may be replaced by the following, after the deletion of the last two lines of p. 957 and the lines 1, 2, 6, 7, 8, 9 of p. 958.

Since, if $k=1$ we infer at once from Lemma 2 that $s_n=o(1)$, we suppose that $k\geqq 2$ and reduce this case to the case $k=1$. When $k\geqq 2$, (7) in p. 958 shows that

$$\sum_{r=0}^{\infty} \Delta^k s_{r-k} x^r = o(1) \,, \qquad x\to 1-0 \,,$$

that is, that the series $\sum \Delta^k s_{r-k}$ is summable (A) to 0. In this series, the nth term $\Delta^k s_{n-k}=o(n^{-k})$, $n\to\infty$, by hypothesis, so that the series is convergent and necessarily to 0. Therefore

$$\Delta^{k-1}s_{n-k+1} = -\sum_{r=0}^{n} \Delta^k s_{r-k} = \sum_{r=n+1}^{\infty} \Delta^k s_{r-k} = \sum_{r=n+1}^{\infty} o(r^{-k}) = o(n^{-k+1}) \,, \qquad n\to\infty \,.$$

By repetitions of this argument (if necessary), we reduce $k\cdots$.

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