CERTAIN GENERALIZED HYPERGEOMETRIC IDENTITIES OF THE ROGERS-RAMANUJAN TYPE (II)

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1. Introduction. Nearly two years ago, Alder [1] established the following generalizations of the well-known Rogers-Ramanujan identities:

$$(1)\qquad \prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-M})(1-x^{(2M+1)n-(M+1)})(1-x^{(2M+1)n})}{(1-x^n)}=\sum_{t=0}^{\infty}\frac{G_{M,t}(x)}{(x)_t}\,,$$

$$(2)\qquad \prod_{n=1}^{\infty}\frac{(1-x^{(2M+1)n-1})(1-x^{(2M+1)n-2M})(1-x^{(2M+1)n})}{(1-x^n)}=\sum_{t=0}^{\infty}x^t\frac{G_{M,t}(x)}{(x)_t}\,,$$

where $G_{M,t}(x)$ are polynomials which reduce to x^{t^2} for $M=2$ and

$$(x)_t=(1-x)(1-x^2)\cdots(1-x^t)\,,\qquad (x)_0=1\,.$$

In a recent paper [6] I gave a simple alternative proof of (1) and (2). We used the result

$$(3)\qquad 1+\sum_{s=1}^{\infty}(-1)^s k^{Ms}x^{\frac{1}{2}s\{(2M+1)s-1\}}(1-kx^{2s})\frac{(kx)_{s-1}}{(x)_s}$$
$$=\prod_{n=1}^{\infty}(1-kx^n)\sum_{t=0}^{\infty}\frac{k^tG_{M,t}(x)}{(x)_t}\,,\qquad M=2,3,\cdots$$

Alder in his paper states that identities involving the generating function for the number of partitions into parts not congruent to 0, $\pm(M-r)(\mathrm{mod}\ 2M+1)$, where $0\leqq r\leqq M-1$, can be obtained by his method and indicates the result for $r=1$.

In the present paper I give a simple method of obtaining the M identities for each modulus $(2M+1)$. In § 4 identities for which $r\geqq\frac{1}{2}M$ have been deduced and in § 5 those for which $r\leqq\frac{1}{2}M$ have been obtained for any r such that $0\leqq r\leqq M-1$. The identities given in § 5 have not been mentioned by Alder. As a corollary, an interesting identity between two infinite series is given.

2. Notations. Assuming $|x|<1$, let

$$(\alpha)_n\equiv(\alpha)_{x,n}=(1-\alpha)(1-\alpha x)\cdots(1-\alpha x^{n-1})\,,\qquad (\alpha)_0=1\,,$$

$$(\alpha)_{-n}=(-1)^nx^{\frac{1}{2}n(n+1)}/\alpha^n(x/\alpha)_n\,,$$

$$x_n=1+x+x^2+\cdots+x^{n-1}\,.$$

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